

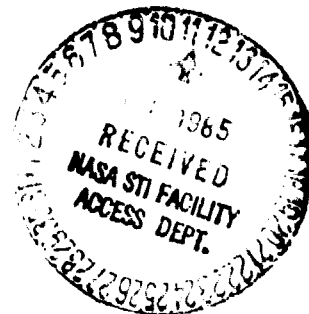
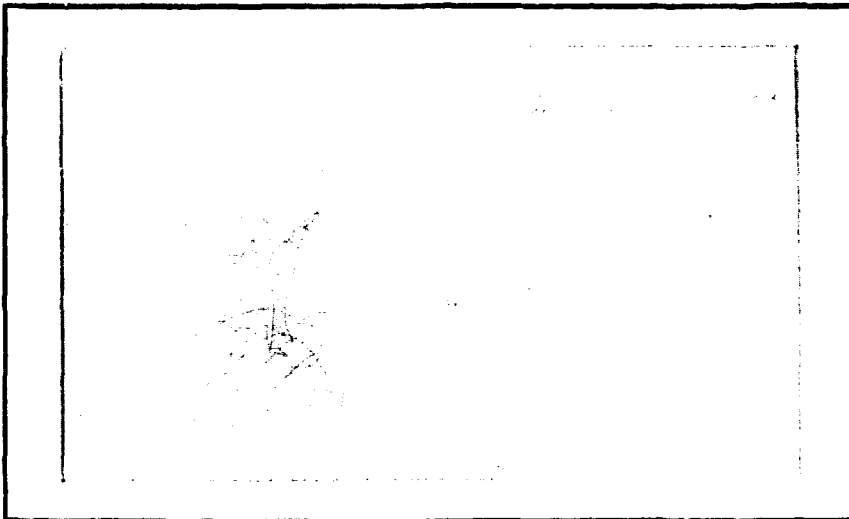
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Strength Modeling Report

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1. Introduction

Strength modeling is a complex and multi-dimensional issue. There are numerous parameters to the problem of characterizing human strength, most notably:

- Position and orientation of body joints
- Isometric versus dynamic strength
- Effector force versus joint torque
- Instantaneous versus steady force
- Active force versus reactive force
- Presence or absence of gravity
- Body somatotype and composition
- Body (segment) masses
- Muscle group involvement
- Muscle size
- Fatigue
- Practice (training) or familiarity

In surveying the available literature on strength measurement and modeling we have attempted to examine as many of these parameters as possible. The conclusions reached at this time point toward the feasibility of implementing computationally reasonable human strength models. The assessment of accuracy of any model against a specific individual, however, will probably not be possible on any realistic scale. Taken statistically, strength modeling may be an effective tool for general questions of task feasibility and strength requirements.

The observations fall into four broad classes:

1. Kinematic and dynamic simulation including mass and inertia of certain body chains, such as the full arm or leg, are mechanically feasible and could be structured around empirical data values for some particular individual or population. Simple forward dynamics (forces from torques) and backward dynamics (reactive forces) may be computable by known methods.
2. Existing strength databases may be made available through computer database query systems and the resulting data interpolated to provide approximate strength data for positions not measured directly.
3. There are a number of strength measuring devices available, and the outputs of all of these appear amenable to computer utilization in strength modeling systems.
4. The graphical display of strength data, whether empirically measured or analytically derived, is quite feasible on present generation graphics devices using the existing

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classes of human body models.

By *simulation* we mean the explicit mathematical modeling of the mechanical structure of some part of the body. There are several ways to build these models. We will examine methods developed for both mechanical engineering linkage analyses and robot manipulator control. Both of these approaches include kinematic (position, velocity, and acceleration) and dynamic (force) computations. It appears that kinematic and dynamics simulations are theoretically and mechanically feasible and could be structured around empirical data values for some particular individual or population.

For an entire body simulation, however, the problem develops new difficulties. Full body muscular dynamics may be formally expressible, but the effective computational complexity of this task is not known. Simple forward dynamics (forces from torques) and backward dynamics (reactive forces) should be computable by known methods, but the construction of an accurate muscle strength and attachment model may be quite formidable. In the next section we examine in detail models of human muscle strength.

2. Muscle Strength Models

Muscular strength is the force or torque that can be exerted within a specified period of time [29] [15]. It is the result of a complex interaction of many internal factors. The major determinants are the muscular system, the skeletal system, and the nervous system. The muscular system is the force generator; it produces tension by contraction of muscle fibers. The skeletal system provides a mechanical framework to transmit the force. The nervous system is a closed-loop system which stimulates muscles to contract. Many other factors, including fatigue, motivation, and position, influence strength. Their effects are difficult to isolate and often complicate efforts to objectively measure strength.

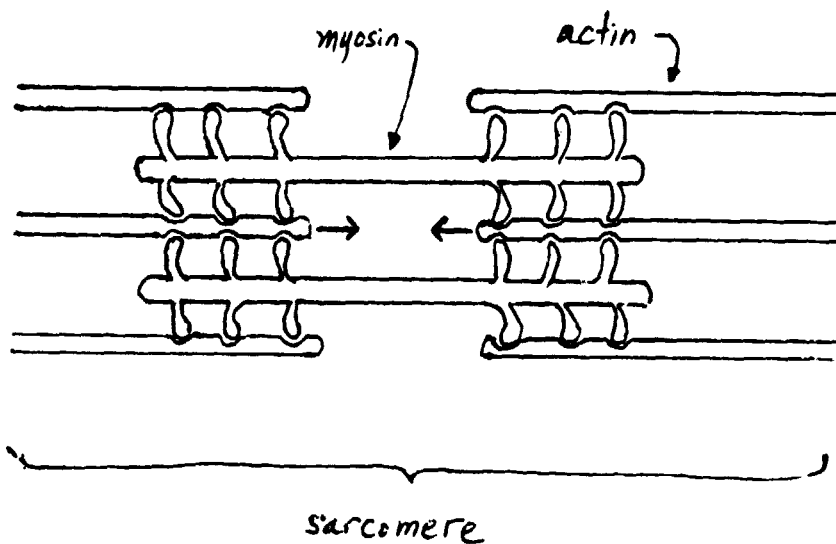
2.1. The Muscular System

There are three types of muscles: *cardiac*, *smooth* and *skeletal*. Cardiac muscle is found only in the heart. Its control centers are located within the muscle. Smooth muscle, also called involuntary muscle, is found in internal organs: the walls of the digestive tract, the walls of blood vessels, the iris of the eye. It is under control of the autonomic (involuntary) nervous system. Skeletal muscle, also called striated or voluntary muscle, is under the control of the somatic (voluntary) nervous system. Skeletal muscles mostly provide force to propel the skeleton. Some skeletal muscles, however, are not attached to bones, e.g. lip muscles. The three types of muscles differ in their microscopic structure but use the same proteins for contraction. In the context of this study, only skeletal muscle is involved, and the term muscle in this report will refer to skeletal muscle.

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The basic contractile unit is the sarcomere (Fig 2-1). It consists of overlapping filaments of myosin and actin protein. Projections on the myosin filaments extend outward to adjacent actin filaments. In contraction, these projections act as ratchets to slide the actin filaments over the myosin filaments. Linear arrangements of sarcomeres make up a myofibril. In turn, bundles of myofibrils form a muscle fiber. Each fiber also contains respiratory organelles and an internal membranous network, the sarcoplasmic reticulum. The sarcoplasmic reticulum reacts to nervous stimulation by releasing calcium ions which are needed for contraction.

Figure 2-1: Contractile unit of skeletal muscle



There are several types of muscle fibers, separated into two main categories: 1) fast twitch, F-type or type II fibers, and 2) slow twitch, S-type or type I fibers. S-type fibers are aerobic and have twice the blood supply (capillaries) of F-type fibers. They are slow to respond to stimulation, but have high endurance. F-type fibers are anaerobic, have a larger diameter and a more extensive sarcoplasmic reticulum than S-type fibers. They respond quickly to stimulation and can produce twice as much tension as S-type fibers, but fatigue easily.

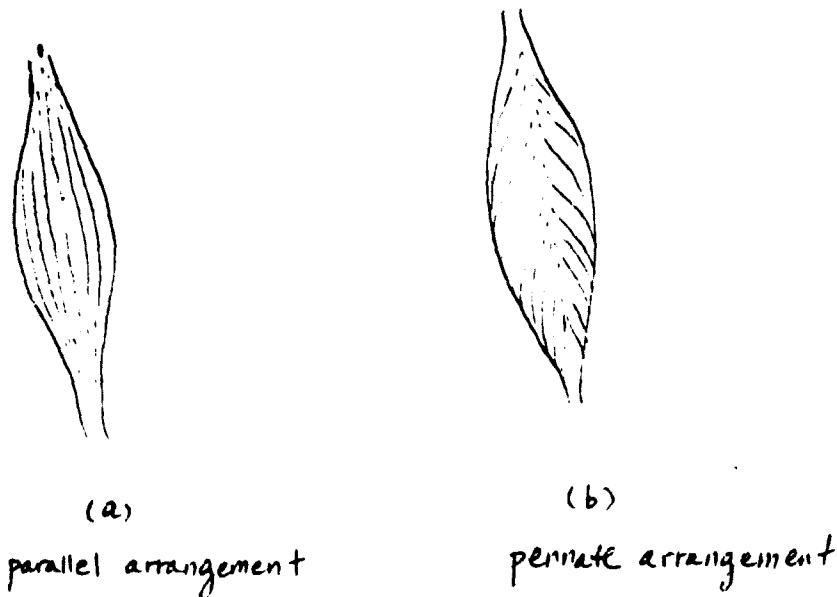
Muscle fibers grow by hypertroph: growth in diameter without growth in the number of cells. Hypertrophy occurs by synthesis of protein [32]. It can be induced by high-intensity exercises such as weight-lifting. Endurance exercises, however, have no effect [9]. Atrophy, the opposite of hypertrophy, occurs from disuse. However, if the muscle is kept at passive tension, atrophy occurs less rapidly [32].

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A motor unit is the smallest set of fibers that can be stimulated at one time. Each unit is made up of only one type of fiber. However, the fibers of a unit may be interspersed with the fibers of another unit. In moderate activities, motor units are activated in sequence to prevent fatigue. As activity increases, more units are recruited at the same time and the frequency of stimulation increases.

Muscle fibers are arranged in two ways, *parallel* and *pennate*. In parallel, the fibers run along the length of the muscle. In pennate, the fibers are at an angle relative to the length of the muscle (Fig. 2-2).

Figure 2-2: Fiber arrangements in skeletal muscle



The power of a muscle depends its cross-sectional area. The larger the cross-sectional area, the more powerful it is. The pennate arrangement allows more fibers and therefore, a greater cross-sectional area. The number of fibers per muscle is fixed at birth, but the diameter of the fibers can be increased by exercise and training. The range of a muscle depends on the length of the individual fibers. Longer fibers have a greater range of movement. Long fibers usually occur in parallel arrangements.

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Each muscle is encased in a net of connective tissue, the *fascia*. At the ends of the muscle, the fascia becomes continuous with tendons. Tendons attach muscles to bone. They increase the range of a muscle and focus its force on a distal point. Fascia and tendons are of similar material. Both add an elastic component to muscles, the fascia in parallel and the tendon in series. These components buffer the muscle in sudden changes in force.

Muscles have distinctive tension-length relationships. Passive tension is caused by loading of the muscle without stimulation. The tension is due to the elasticity of the material and is negligible until the muscle is stretched to its rest length, l_0 . Usually, this rest length is 125% of its disinserted length [9]. Active tension is caused by loading with stimulation. Subtraction of the passive tension-length curve from the active tension-length curve results in the contractile force curve (also called developed tension curve and extra tension curve). The contractile force is tension due solely to active contraction of the muscle fibers. The contractile force curve peaks at the rest length and decreases at either side.

Bourne [9] and Wilhelms [54] both stated that active muscle tension also peaks at the rest length. Wilhelms further explained that at lengths less than or greater than l_0 , contacts between the myosin filaments and the actin filaments are less than optimal and so produce less tension. This explanation, however, applies only to the contractile force curve. They did not take passive tension into account. Clarke and Ralston [15] [43] have obtained experimental data which show that beyond l_0 , passive tension can offset the decrease in contractile force (Fig. 2-3). For the human biceps, there is a local maximum at l_0 .

2.2. The Skeletal System

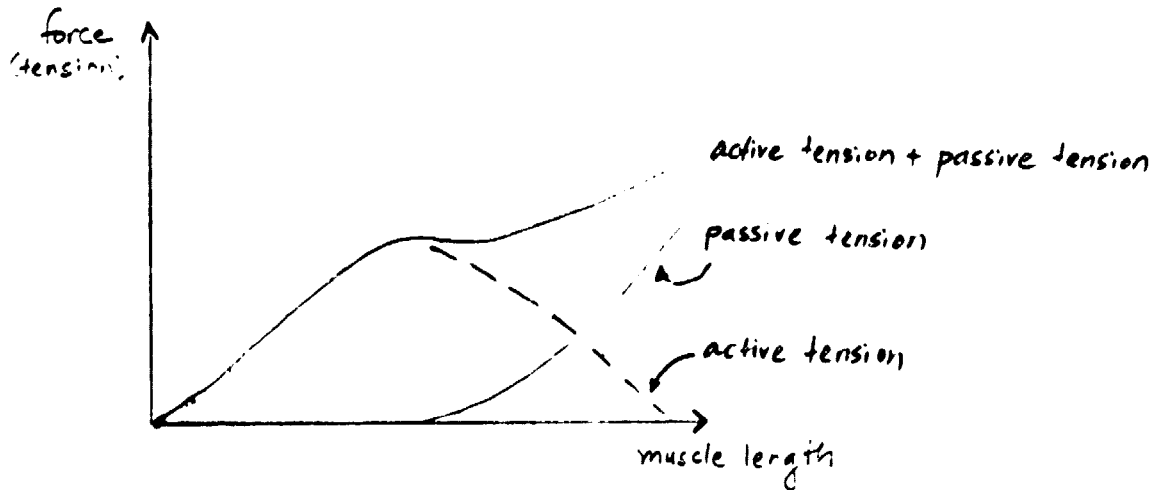
The human skeleton is composed mostly of bones. It is divided into two main categories: the *axial skeleton* and the *appendicular skeleton*. The axial skeleton consists of the skull (cranium), the vertebral column, and the bones of the chest (thorax). The appendicular skeleton consists of the limbs, the pelvic (hip) girdle, and the pectoral (shoulder) girdle.

Bones meet at joints. Each joint is specialized for a certain type or types of movement. Thus, joints limit the degrees of freedom [53].

Ligaments, which are of the same material as tendons, connect bones. They are passive structures and are essential for the control and stability of various joints in normal activities. However, detailed mechanics of the ligaments in various position of flexion-extension is still controversial. This is particularly true, for example, of the anterior and posterior cruciate of the knee which are relatively inaccessible. Structural orientation of the fibers of muscle/tendon shows that the tendons have almost completely parallel alignment, which makes the tendon well

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Figure 2-3: Isometric length-tension relationship



suited for withstanding high tensile loads. However, the fibers of the ligaments have less consistent structural orientation which varies in different ligaments depending on their function.

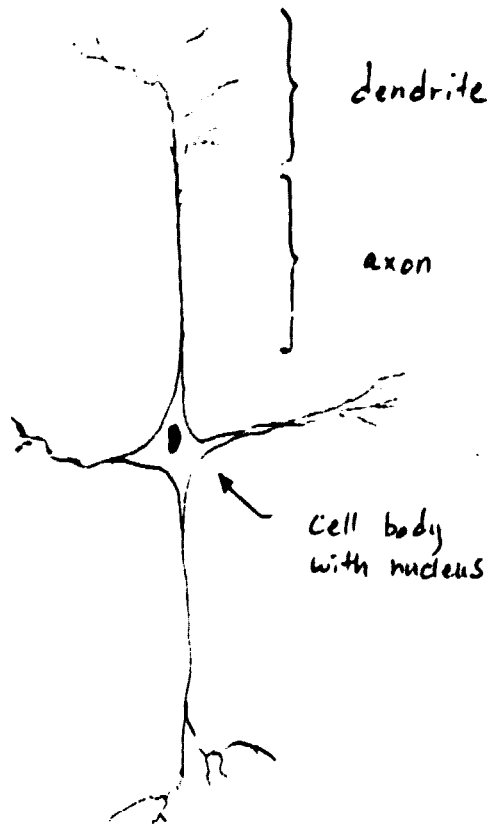
2.3. The Nervous System

Internally, skeletal muscles are stimulated by the somatic or voluntary nervous system. Each nerve cell is called a *neuron*. A group of neurons form a nerve. In general, neurons have a *dendrite* (tree) region, a *cell body* and an *axon* (Fig. 2-4). The dendrite is a highly branched region which receives stimuli, from the environment or from other neurons. The cell body contains the nucleus and the respiratory organelles. The axon is the trunk of the neuron. It allows the neuron to spread a signal far away from the cell body. Each axon ends in numerous swellings called *synaptic knobs*. The knobs communicate, or *synapse*, with other neurons or with effectors such as muscles or glands [53].

Neural impulses are spread both electrically and chemically. Along the axon, the impulse is spread by electrical charges. At the synaptic knobs, the electrical signals cause the knobs to release chemicals called neurotransmitters. These neurotransmitters activate the next neuron or effector.

Each motor unit has associated with it an axon. Together, they form a *neuromotor unit*. The axon of a neuromotor unit sends a filament with synaptic knobs to each fiber in the unit.

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Figure 2-4: Neuron



Motor units with fewer fibers can achieve more precise and rapid movements [9].

2.4. Terminology

2.4.1. Body Position

The human body is described by a set of three orthogonal planes: the *sagittal*, the *transverse* and the *frontal* planes. The *sagittal* plane divides the body into left and right portions. The *medial sagittal* plane divides the body into symmetric halves. The transverse plane, also called the horizontal plane, is parallel to the ground when the body is upright. The frontal plane, also called the coronal plane, divides the body into front and back.

Medial refers to the midline of the body. *Lateral* refers to any point far from the median line. *Anterior* and *ventral* refer to the front of the body; *posterior* and *dorsal* refer to the back. *Superior* and *cranial* refer to the head end of the body; *inferior* and *caudal* refer to the feet end. *Proximal* and *distal* are relative terms used in reference to limbs along their long axis. *Proximal* indicates a point near the attachment of the limb, and *distal* indicates a point away from the attachment.

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2.4.2. Joint Movements

Flexion decreases the angle between bones; movement is usually toward the ventral surface. Extension increases the angle, and movement is toward the dorsal surface. Abduction is movement away from the midline of the body. In the case of the fingers, it is movement away from the midline of the hand through the middle finger. Adduction is movement toward the midline of the body; with fingers, it is movement toward the midline of the hand. Rotation is turning of the body about its long axis. Medial or internal rotation causes the ventral surface to turn toward the body midline. Lateral or external rotation causes it to turn away from the midline [24].

2.4.3. Origin and Insertion

Origin and insertion are terms applied to the ends of a muscle. The origin is the end that is relatively fixed in movement. In most cases, it is the attachment point closer to the midline of the body. The insertion is the end that is relatively mobile in movement. The terms, however, are not strictly defined. The actions of the origin and insertion can often be reversed.

2.4.4. Muscle Groups

A muscle group is a set of muscles which perform similar functions. They are often wrapped together by a net of *deep fascia* [24], which is of the same material as fascia. In general, four muscle groups are needed to carry out a task:

- agonists - the primary movers.
- antagonists - oppose the action of the agonists and must relax for the agonists to be effective.
- fixation muscles - fix the base upon which movement by agonists is carried out.
- synergists - nearby muscles which aid the agonists; if the agonists degenerate, the synergists frequently take over their functions.

2.4.5. Isometric, Isotonic and Isokinetic Contraction

There are several well defined categories of contraction: *isometric*, *isotonic*, and *isokinetic*. Isometric contraction is contraction in which the length of the muscle remains constant. Isotonic contraction is contraction in which the external force (torque) is constant. *Concentric* contraction is isotonic contraction in which the length of the muscle decreases; *eccentric* contraction is contraction in which the length increases. Isokinetic contraction is contraction in which the (angular) velocity of contraction remains constant.

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2.5. Factors Affecting Muscular Strength

2.5.1. Motivation

One of the most perplexing problems in muscle strength testing is obtaining maximum voluntary contraction, *MVC*. *MVC* is the maximum strength a person can exert without injury. In practice, however, subjects psychologically set a safety limit that is often lower than the actual. Nelson [37] has shown that motivation can alter the perception of that safety level. In a study of 250 men, he found that certain instructions elicited higher strength efforts. These instructions tended to challenge or boost the ego. They included comparisons to the results of other tests.

2.5.2. Position

Body position affects the length of muscles. From the tension-length relationship, isometric strength depends on the muscle length. Clarke [15] has shown that changing body position can also inactivate certain synergic muscles. By rotating the humerus inward during shoulder adduction, the biceps can be eliminated and result in a decrease of measured strength.

2.5.3. Fatigue

Physiological fatigue is "a state in which the activity of a muscle decreases despite continuous stimulation but returns to normal after rest" (J. Scherrer). The general procedure in fatigue tests is to have a subject repeatedly pull a fixed load F over a distance l and rest. The repetition is continued at a set pace until exhaustion. Mosso [36] made the first studies of muscular endurance. He is credited with inventing the ergograph, which records the displacement of load with time. In his studies, he allowed the distance l to vary during work. He took exhaustion to be the point at which displacement was no longer detectable and obtained uniformly decreasing "curves of fatigue." Clarke [15] followed this method but found that at certain loads and rates of work, the fatigue curves did not steadily decrease. In these curves the decline was not smooth.

Monod and Scherrer [34] used a different approach. They fixed both F and l , and designated exhaustion to be the point at which l begins to decrease (Fig. 2-5). They varied the work rate and found that the maximum amount of work done before exhaustion was inversely proportional to the work rate :

$$W_{lim} = \frac{t_{lim}}{P}$$

where P is the work rate; W_{lim} is the maximum work before exhaustion; t_{lim} is the length of time in which work was done. They further derived a relationship between W_{lim} and t_{lim} (Fig.

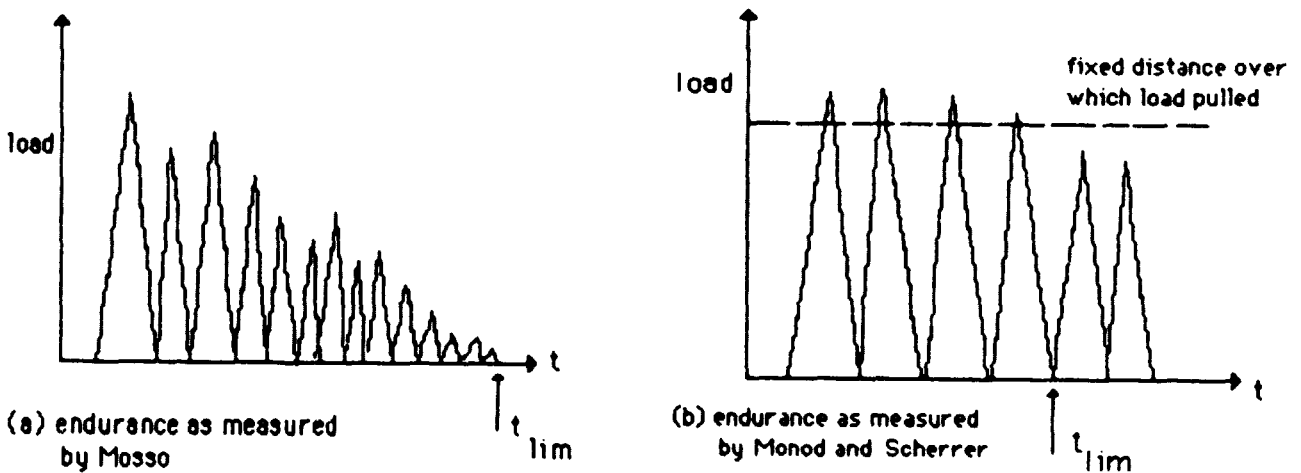
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2-6):

$$W_{lim} = a + bt_{lim}$$

where a and b are physiological parameters related to the muscle's energy reserve and rate of energy reconstitution, respectively. At $P = b$, the rate of energy expenditure is equal to the rate of energy reconstitution. The value b , then, is the highest rate at which work can be done continuously without exhaustion.

Figure 2-5: Measuring endurance



2.6. Previous Studies in Determining Strength

2.6.1. Direct Measurements

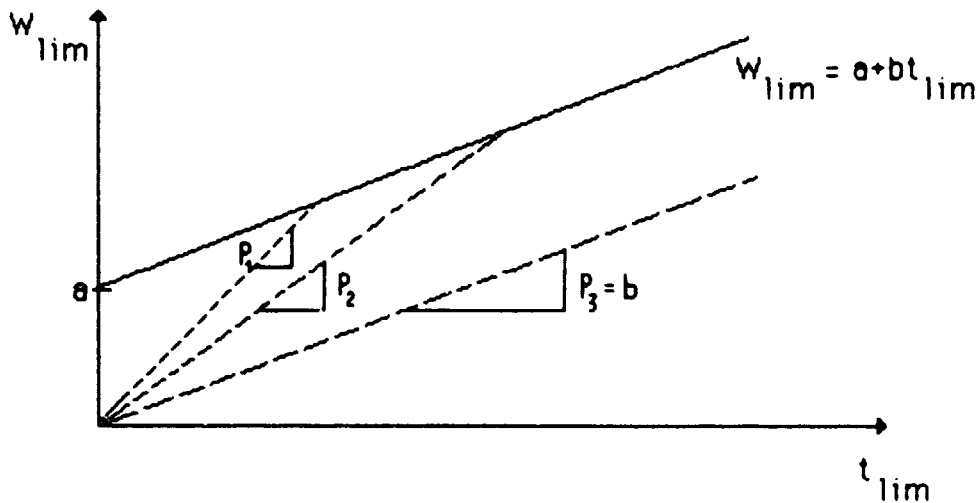
Subjective:

Most clinical assessments of muscular strength are subjective. The tester usually resists the movements of the subjects and grades strength on a scale of normal to trace. This method is limited by the strength of the tester, and requires that the person be very experienced. However, it is adequate for most clinical purposes as the aim is to determine *loss* of strength rather than absolute strength.

There are several grading systems, differing in their definitions of each grade. The most popular system is the Lovett Method [31].

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Figure 2-6: Relationship between work and work rate



Objective:

Equipment to measure strength is based on either the cable tensiometer or the strain guage. The tensiometer determines strength by the amount of tension applied to a cable. The strain guage consists of several guage rings. Strength is determined by the change in electrical resistance of the rings as force is applied to the strain guage. Clarke [15] felt that the strain guage was too sensitive to temperature and that deformations disappear too slowly. Wakim [52], on the other hand, felt that the internal resistance (friction) of the cable tensiometer was too high, and that it may increase with increasing force. Most instruments use the strain guage and are calibrated prior to each measurement to account for any changes due to temperature or permanent deformation.

The problem with objective tests lie in the procedure. At present, there is no standard method of obtaining strength data. Data from different studies often cannot be compared because they were taken under different circumstances, including the type of instructions given, the body position, and the amount of rest given between trials.

2.6.2. Electromyography

Electromyography, EMG, detects electrical activity (stimulation) in the muscles. Its use is limited to those muscles that are near the surface. Moreover, it is a qualitative index of muscular strength. Cnockaert et al [16] attempted to quantify EMG measurements, but their results have not been convincing.

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2.6.3. Cinematography

Cinematography is most often used in gait analysis to determine indirectly force and velocity of movement. Velocity and acceleration are derived from displacement of markers placed on the skin. Force is then calculated as acceleration x mass. The errors in this method are due to:

- movement of the markers relative to the skin
- measurement errors
- calculation errors
- errors in the anthropometric data on the mass of each limb

2.6.4. Chaffin's Biomechanical Model

Chaffin [11] developed the computerized Static Sagittal Plane (SSP) model to analyze static and pseudo-static lift efforts in the sagittal plane; he recently extended it to 3-dimensional lift tasks. The SSP model is a system of 7 links representing the body. It requires as input (1) the lift load, taken as acting on the center of gravity of the hand and (2) the angle between each link in the lift posture. From this information, the model calculates the force required from each link to stabilize and maintain the position. Included in this model is an analysis of the compression on the spine and the abdominal pressure that is developed during lifts. The results indicate that lifting capacity is not limited by muscular strength but by compression on the spine. Abdominal pressure alleviates compression and enables a greater lift.

2.6.5. Physical Characteristics

Several attempts have been made to correlate muscular strength to physical characteristics. Lamphiear and Montoye [30] studied the relationship between body size and isometric strength. They measured isometric grip and upper arm strengths of 2,713 subjects and correlated the measurements to 12 size variables. They found that most of the variance in strength could be accounted for by only 5 variables:

- height
- weight
- biacromial diameter
- arm girth
- triceps skinfold thickness

They derived sex and age specific equations based on these variables to predict strength.

Hosler and Morrow [27] assessed the role of gender in determining strength. They

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measured arm and leg strengths of 87 men and 115 women. Using stepwise analysis, they correlated the measurements to body size, body composition (fat content) and gender. In the first analysis, gender was entered as the only variable and found to account for 60-74% of the variance in strength. In a second analysis, the effects of body size and composition were first eliminated by entering them into the analysis. Inclusion of gender at this point accounted for an additional 1-2% of variance. The variance in gender was absorbed by the other two variables. Differences in gender, then, was attributed mostly to size and composition.

2.7. Joint Force Calculations

Internal forces and moments at a joint are difficult to measure directly. The force system has to keep the entire structure in static equilibrium at all times. The intersegmental force and moment resultants at the joints are determined approximately by modeling the body or parts thereof as a system of rigid links. Joint (inverse) force calculations involve calculating the internal joint reactions for a particular body position due to external forces. These joint resultants are then distributed to the muscles and ligaments using a simplified representation of joint anatomy. Inspection of the force distribution can show whether the body can possibly maintain static equilibrium at a specified body position. The maximum stress can be calculated from the force distribution and the cross sectional area of the muscles. A joint cannot maintain a particular position when the maximum stress is greater than the maximum allowable stress of the muscles.

2.7.1. Coordinate Transformations

The body is modeled as a system of rigid bodies kept in equilibrium. A global coordinate system are related to the external forces. The coordinates of the joints are chosen with respect to a set of local axes which are located at the joints. The length and weight of the body segments and their respective center of gravity are estimated from anatomical literature.

In order to calculate the local joint coordinate with respect to a global axis, homogeneous transformation matrices are required (Fig. 2-7). For example, the change in coordinates of a point on body 3 resulting from rotation about the Y axis is given by:

$$[X_G \ Y_G \ Z_G \ 1]^T = [T_{\theta 1}][T_{c1}][T_{\theta 2}][T_{c2}][T_{\theta 3}][T_{c3}][X_4 \ Y_4 \ Z_4 \ 1]^T$$

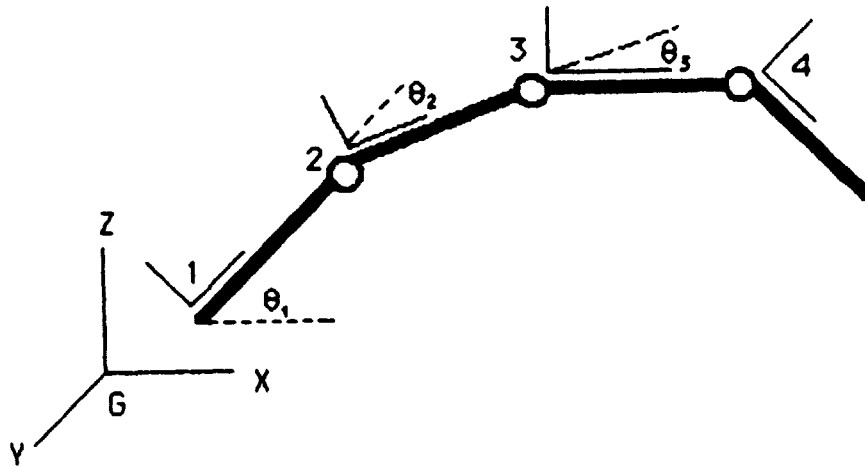
where $[T_{\theta}]$ is a rotational matrix and $[T_c]$ is translational matrix.

2.7.2. Equilibrium Equations

Equilibrium equations for the total body are used to determine the external reactions such as reactions due to contact with a surface. The general equations of force and moment equilibrium are written as:

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Figure 2-7: Coordinate transformation axes.
 θ are angles between coordinate pairs.



$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

$$\Sigma M_x = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = 0$$

2.7.3. Determinacy

Application of the equilibrium equations are limited to body positions and end conditions which render the problem determinant. The end conditions, that is, the condition of the body when it is in contact with an external surface are assumed to produce no moment at that location. No moment is produced, only forces in the x, y and z direction, when a foot, knee or elbow makes contact with a surface. Zero moment also occurs when a hand grips a restrained object. Distributive forces occur when a body is in a seated position. Point contact is assumed and the distributed forces are resolved into a resultant force. Examples of determinant postures for a body are given in fig. 2-8 and 2-9.

Indeterminacy is when the number of unknown forces and moments are greater than the number of equations. A possible occurrence of indeterminacy is when the body is additionally restrained by external forces such as a belt (Fig. 2-10). In this case, two variables are introduced and only one additional equation is added to solve the force system.

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Figure 2-8: Body with determinant force system.

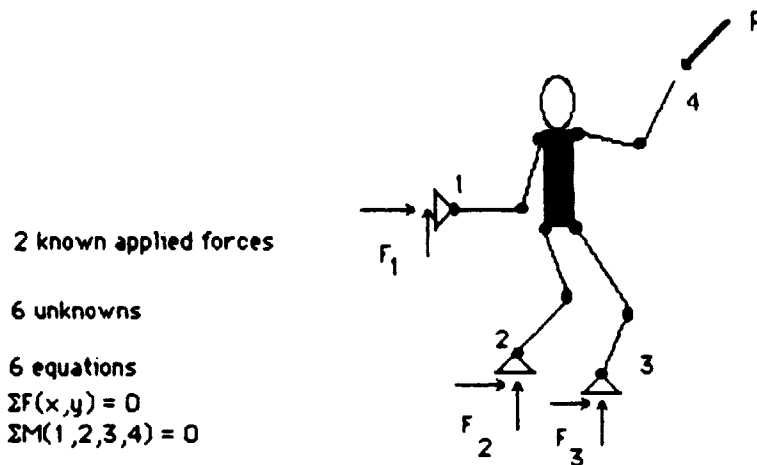
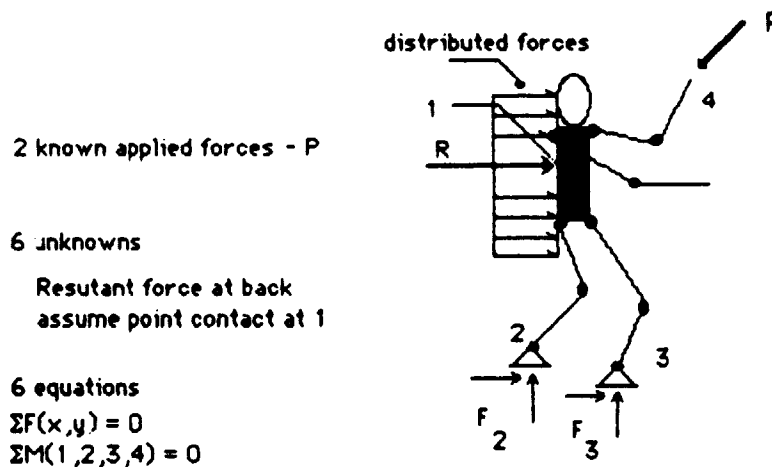


Figure 2-9: Determinant force system. Distributive forces resolved into resultant force.



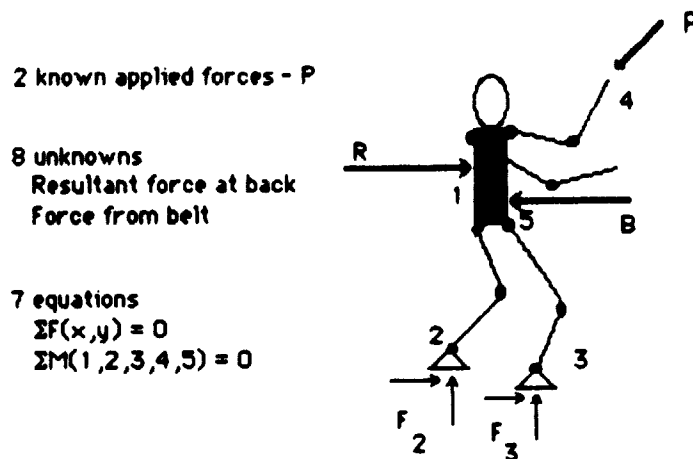
Solution of an indeterminate problem can be achieved by using stiffness coefficients which relates the displacement to the forces. The use of this relation, called the compatibility condition, reduces the order of indeterminacy by the number of stiffness equations which can be applied. Indeterminate problems are not considered here since it requires the material properties of the external surface which a body is in contact.

2.7.4. Joint Forces

Once the external forces are computed for a body orientation the forces and moments at each joint can be computed by applying the equilibrium equation for each segment. Paul [39] equates this to the transformation of forces and moments between coordinate frames. The

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Figure 2-10: Indeterminant force system. Addition of belt restraint.



transformation from the present coordinate frame to a frame c is represented by the simple relationship:

$${}^cM_x = n \cdot ((F \times p) + M)$$

$${}^cM_y = o \cdot ((F \times p) + M)$$

$${}^cM_z = a \cdot ((F \times p) + M)$$

$${}^cF_x = n \cdot F$$

$${}^cF_y = o \cdot F$$

$${}^cF_z = a \cdot F$$

where

c = indicates the coordinate frame the force/moment are transformed toward

F = generalized force vector

M = generalized moment vector

n, o, a, p = first, second, third and fourth columns of a transformation matrix.

The equilibrium equations in this form are useful for efficient computer calculations. (see Appendix I for computer program)

2.8. Determining Force Distribution: Optimization Models

The large number of muscles and ligaments involved usually renders the problem of finding the resultant forces in the muscles indeterminate because there are more unknowns than equations. There are two major approaches to solving this indeterminate problem. One approach, called the "reduction" method, utilizes EMG data or other justifications to reduce the number of unknowns so that the problem is determinate. Paul (1955) [38] first used this

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method to get the force distribution at the hip. The reduction method was applied by Morrison [35] to the knee joint under dynamic conditions and by Chao et al. [13] to distribute joint resultants under static conditions. This method is primarily useful in instances where the actual joint anatomy is very simple or functions simply.

An alternate procedure of solving the distribution problem, the "optimization" method, was introduced by Seireg and Arvikar [47] and Penrod et al. [41]. In this method, it is assumed that the distribution process occurs in such a way as to optimize some kinetic property called the objective function. The proper objective function is not known *a priori*, so the appropriateness of the function chosen must be established indirectly on the basis of the results obtained. Examples of the objective function are the minimization of forces and/or moments, minimization of mechanical energy, and minimization of stress.

2.8.1. Muscle Model

Biomechanicians have modeled the lines of action along which the muscles act in two basically different ways. These two methods are called the straight line method and the centroid line method. The straight line method requires that the approximate points of muscle attachment be determined on body segments proximal and distal to the given joint, and then assumes the force transmitted by the muscle acts along the straight line connecting the two points. The centroid line model requires the locus of transverse cross-sectional centroids be established for the muscle in a variety of joint configurations, and then assumes that the force transmitted by the muscle, at any point on this three-dimensional locus, is tangent to the centroid line at that point.

Although the centroid line model correctly represents the line of action for a muscle as a curved path in the joint neighborhood, it has a number of disadvantages. First, its use requires the collection of a large amount of data to represent a single muscle in only one configuration. Second, a transverse cross-section of a muscle is difficult to define in a meaningful way. Problems occur when they have broad attachments or have an unusual shape. Third, the model is not easy to use when the joint configuration changes. Fourth, curved centroid lines obtained from section cadaver specimens may not accurately represent *in vivo* data.

2.8.2. Coordinate Transformations

The system of rigid bodies is kept in equilibrium by the pull on the muscles or ligaments. The muscle forces are assumed to be directed along lines joining the corresponding points of origin and insertion on the skeletal system. The coordinates of the points of origins and insertion are chosen with respect to a set of axes which are located at the joints. The location of the points of origins and insertions, the weight of the body segments, and their respective

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centers of gravity are estimated from anatomical literature.

2.8.3. Equilibrium Equations

The general equations of force and moment equilibrium for each segment of the body are written as:

$$\begin{aligned}\sum F_x &= \sum F_i l_i + F_x = 0 \\ \sum F_y &= \sum F_i m_i + F_y = 0 \\ \sum F_z &= \sum F_i n_i + F_z = 0 \\ \sum M_x &= \sum M_{xi} + M_x = 0 \\ \sum M_y &= \sum M_{yi} + M_y = 0 \\ \sum M_z &= \sum M_{zi} + M_z = 0\end{aligned}$$

where

F_i = tensile force in the muscle i ,

M_{xi}, M_{yi}, M_{zi} = moment of force in muscle i about the respective axes,

l_i, m_i, n_i = directional cosines for the muscle i calculated from the coordinates of the point of origin and point of insertion,

M_x, M_y, M_z = moment about the respective axes due to all forces other than muscle forces acting on each body as well as any additional joint moments carried by the ligaments, M_{AX} , M_{AY} and M_{AZ} at joint A.

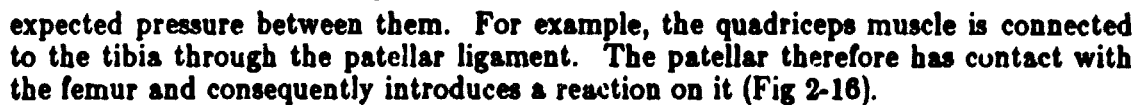
Free body diagrams for the analysis of the lower extremities are shown in Fig 2-11, 2-12, 2-13, and 2-14. The seven segments of the lower extremities would yield 42 equilibrium equations. With 31 muscles on either side of the sagittal plane, 3 joint reaction components along the three reference axes at each of the six joints, 3 moment components at each joint, and 3 patellar reactions on each side, the total number of unknown variables is 104. Therefore, the net number of unknown variables is 62.

2.8.4. Musculo-Skeletal Model

The main considerations for developing the model are:

1. The muscles are assumed to produce tensile forces only.
2. The action of each muscle is represented by one or more lines to simulate the capabilities of the muscle in three dimensional space. For example, the adductor magnus muscle has two parts— the adductor part and the extensor part. Consequently the muscle is represented by two lines as indicated in Fig 2-15.
3. Whenever a straight line representing a muscle is interrupted by some interposing structure, the direction of the line is changed to wrap around the the structure and a resultant reaction is assumed on both the muscle and the structure to simulate the

Figure 2-11: Force model for the pelvis [47]

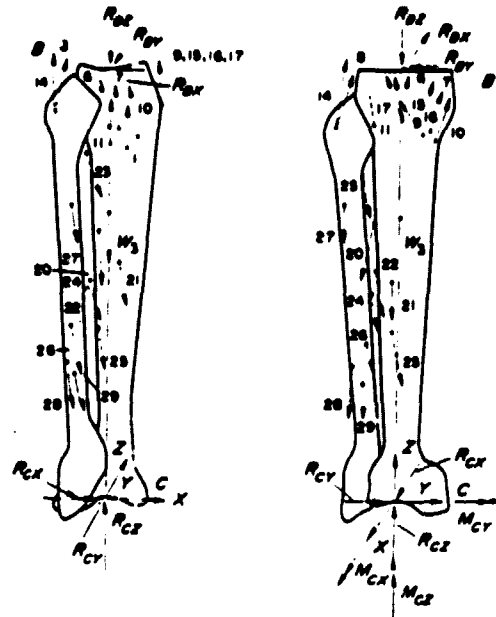


2.9.1. Optimization Functions Based on Forces and Moments

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Figure 2-13: Force model for the fibula [47]



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Figure 2-14: Force model for the foot [47]

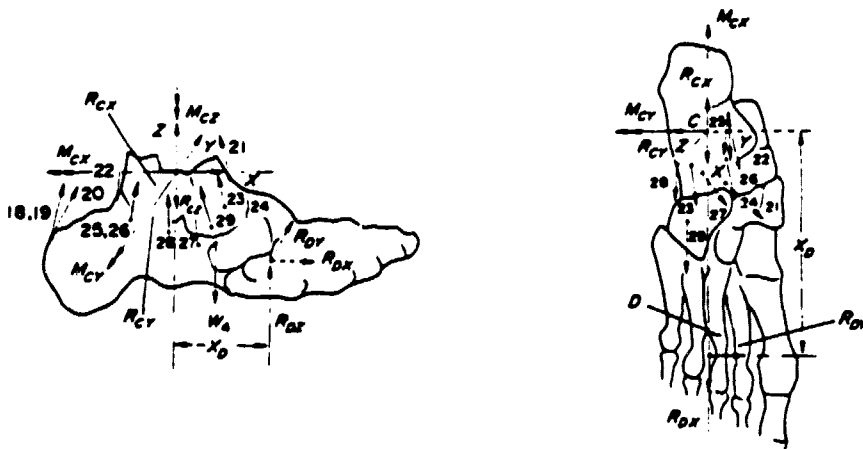
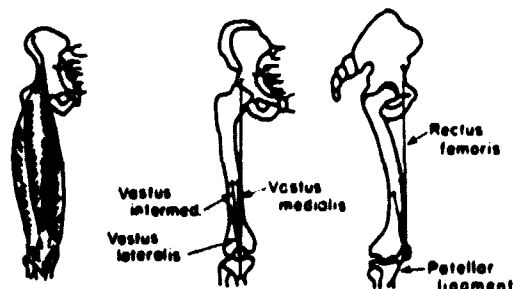
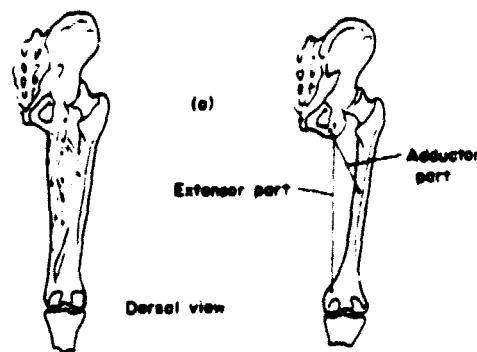


Figure 2-15: Muscle model for the quadriceps [47]



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Figure 2-16: Muscle model for the adductor magnus [47]



- minimization of forces in the muscles:
objective function $U = \sum F_i$
- minimization of the work done by the muscle to attain the given posture, that is, minimize the product of the muscular tension and its elongation or contraction:
objective function $U = F_i |\Delta L_i|$
where $|\Delta L_i|$ is the magnitude of muscle extension
- minimization of the vertical reactions R_{AZ} , R_{BZ} , R_{CZ} at the three joints A, B, and C respectively:
objective function $U = |R_{AZ}| + |R_{BZ}| + |R_{CZ}|$
- minimization of the moments carried by the ligaments at the three joints:
objective function
 $U = \sum M$
 $= |M_{AY}| + |M_{BX}| + |M_{BY}| + |M_{BZ}| + |M_{CX}| + |M_{CY}| + |M_{CZ}|$

These objective functions and the equations of equilibrium are linear and, therefore, can be formulated as a linear program and a unique solution is obtainable by the simplex technique.

The model was applied for static cases where the body was standing and leaning forward or backward. The plots of the theoretical results and the experimental verification by EMG of the gastrocnemius and semitendinosus muscles are given in Figs. 2-17 and 2-18 for the leaning posture. The figures serve as a guide for determining the correlation between the theoretical results based on the selected optimization function and the measured muscle response. From the figures it was determined that the best set of criteria was of the form $F + k M$. Table 2-1 gives a summary of different weighting factors k in all the studies. The table suggests that such a criterion with k greater than 4 is applicable to all case [47] [46].

2.9.2. Optimization Functions Based on Stress

In Crowninshield, et al. [18] investigation of the human hip during level walking and other activities imposed an upper bound constraint on the magnitudes of the muscle forces during the distribution process. This is to ensure that possibly unreasonable large forces in the single most advantageous muscle will not be predicted. The linear objective function is of the form

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Figure 2-17: Theoretical and experimental results plotted on independent scales for gastrocnemius [46]

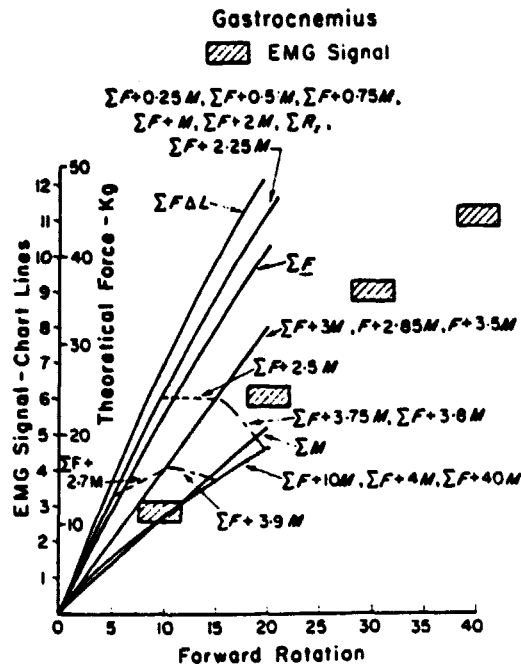
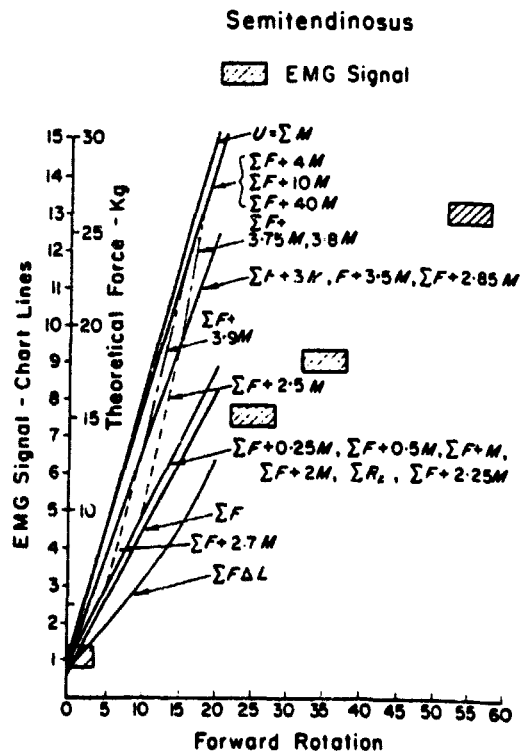


Figure 2-18: Theoretical and experimental results plotted on independent scales for semitendinosus [46]



$$U = \Sigma [F_i/A_i],$$

where A_i is the physiological cross-sectional area of the i -th muscle. Although not precisely

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Table 2-1: Evaluation of feasibility of different criteria [47]

Criteria	Leaning forward	Leaning backward	Stooping
$F + 0M$	✓	×	×
$F + 0.25M$	✓	×	×
$F + 0.75$	✓	✓	×
$F + M$	×	✓	×
$F + 1.25M$	×	✓	×
$F + 1.5M$	×	✓	×
$F + 2M$	×	✓	✓
$F + 3M$	×	✓	✓
$F + 4M$	✓	✓	✓
$F + 5M$	✓	✓	✓
$F + 10M$	✓	✓	✓
$F + 20M$	✓	✓	✓
$F + 40M$	✓	✓	✓
M	✓	✓	✓

✓ possible.
 × not feasible.

defined, the cross sectional area is generally taken to be the muscle's volume divided by its length. The constraints imposed on the unknown muscle forces during the optimization process are that they must be non-compressive, and that they must not exceed a maximum allowable value that is proportional to the physiological cross-sectional area. The required constraint equation is

$$[F_i/A_i] \leq a$$

where a is the maximum allowable tensile stress in the muscles.

The magnitude of the upper bound muscle stress constraint a affects the size of the admissible solution space and, therefore, affects the solution to the distribution problem. The smallest value of a for which the solution space is not empty is denoted by a_c . It was found experimentally that by choosing a that is equal to $1.2 a_c$, a physiologically reasonable solution was obtained.

2.9.3. Optimization Based on Endurance

Fick (1910) and others reported that individual muscle force exertion capabilities can be related to muscle cross-sectional size through a constant of proportionality with the units of stress [23]. Don et al. (1979) showed that the endurance relation was related to the muscle's exertion capabilities and, therefore, endurance was related to stress [20].

Based upon the endurance properties of the muscles, Crowninshield and Brand's [19] objective function was

$$U_n = \sum [F_i/A_i]^n$$

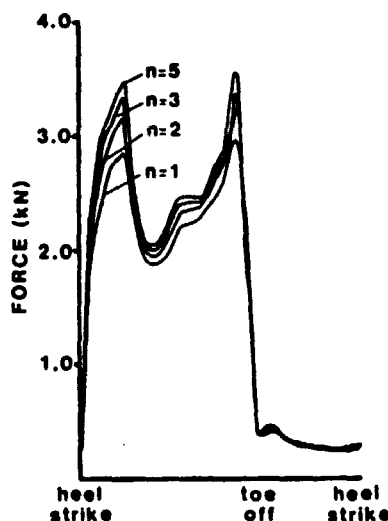
where the appropriate power of n is not known. The actual value of n may vary between

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individual subjects and between individual muscles. Muscle forces predicted in this manner will tend to keep the individual stresses low which will coincide with achieving maximum endurance. Also the fact that the maximum stress that the muscle is capable of, 0.4 to 1.0 MN/m^2 , was used as the upper bound constraint for stress.

Crowninshield and Brand applied the optimization function to the modeling of force prediction in locomotion. Individual muscle forces were predicted incorporating various values of n . The patterns of muscle force prediction are not very sensitive to small change in n as shown in Fig. 2-19. The use of a power of 2.0 may be adequate and offers the advantage of permitting the use of quadratic programming in place of more general nonlinear programming. Crowninshield pointed out that the criterion of maximum endurance might be reasonable for an activity such as walking at a comfortable pace when endurance is great; it might not be reasonable of other activities such as climbing stairs. In such case, the body might point to a different criteria [19].

Figure 2-19: Comparison of Optimization Function for different values of n [19]



2.9.4. Comparison of Optimization Functions

The work by Crowninshield was concerned with the investigation of forces in a quasi-static condition. The forces were calculated for situations when the subject was in locomotion. An et al. [3] compared the various optimization functions for a static situation (at the elbow joint). The optimization functions that were compared are:

1. minimization of forces: $U = \sum F_i$ and $U = \sum F_i^2$
2. minimization of stress: $U = \sum [F_i/A_i]$, and with an upper bound muscle stress constraint: $U = \sum [F_i/A_i], F_i/A_i \leq a$
3. minimization of endurance: $U = \sum [F_i/A_i]^2$

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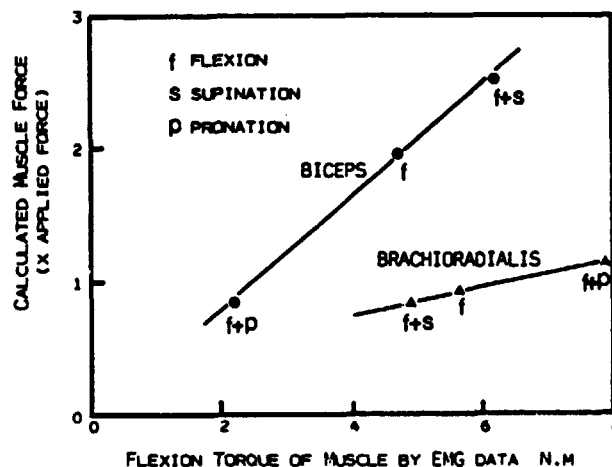
By using the summation of muscle force and summation of stress as the minimizing criteria, only one muscle was predicted to carry an applied force at the elbow joint. The minimization of the nonlinear combinations of the stress with an upper bound provided a more evenly distributed muscle system to carry the applied load. Table 2-2 gives a comparison of the optimization methods and Fig.2-20 compares the theoretical results with EMG data [3].

Table 2-2: Comparison of optimization method for muscle and joint force determination [3]

	$\text{Min } \Sigma F_i$	$\text{Min } \Sigma S_i$	$\text{Min } \Sigma F_i^2$	$\text{Min } \Sigma S_i^2$	$\text{Min } \sigma; S_i \leq \sigma$
F_i/p					
BIC	—	—	1.4	2.5	1.8
BRA	—	9.5	1.0	4.6	2.9
BRD	4.3	—	2.3	0.4	0.6
FCR	—	—	1.5	0.1	0.8
ECRL	—	—	0.9	0.4	0.9
ECRB	—	—	0.8	0.2	0.7
ECU	—	—	0.4	0.1	0.6
R/P	4.0	8.6	6.8	6.9	6.5
(deg)	9.7	66.3	24.0	62.0	41.3

^(a) Load applied at the distal ulna when elbow is in 90 deg of flexion. Only flexion-extensional moment equilibrium equation is considered.

Figure 2-20: Comparison of theoretical results with those from EMG experiments. The muscle forces of biceps and brachioradialis muscles are calculated by the optimization of stress and a upper bound of stress for weight lifting at various forearm position [3]



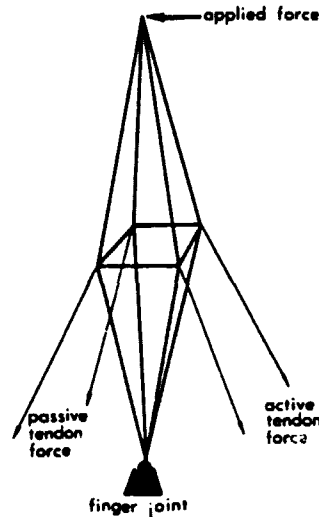
2.10. Antagonistic Muscles

The force system modeled did not include antagonist muscles. Antagonistic muscles produce counterbalancing tensions for the purpose of reducing subluxation forces at the joint, which may cause excessive stretch of the ligamentous structure surrounding the joint. Under this condition, the compressive force at the joint is increased, which will also enhance stability

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(Fig.2-21).

Figure 2-21: Pylon concept for tendon and joint function



2.11. Inclusion of Ligamentous Forces

Ligaments cause nonlinear and coupled load-displacement characteristics at the joint. When the joint is displaced, the ligaments are stretched and develop forces which resist the motion. Shear loads developed during a drawer test are transmitted across the joint line. The ligamentous tensile forces also combined with the bone compressive force permit transmission of moments. Since the ligaments are arranged in parallel, the forces and moments they transmit are additive. Thus the total reaction load is the sum of that due to the individual ligaments and the contact force, as shown in Fig.2-22. The coordinate system that was assigned for the musculo-skeletal model is also used in the modeling of a joint to predict ligament forces .

The joint is modeled by a 12 x 12 beam element stiffness matrix. The force-displacement relationship is given by

$$\{F\} = [k]\{\Delta\}$$

where

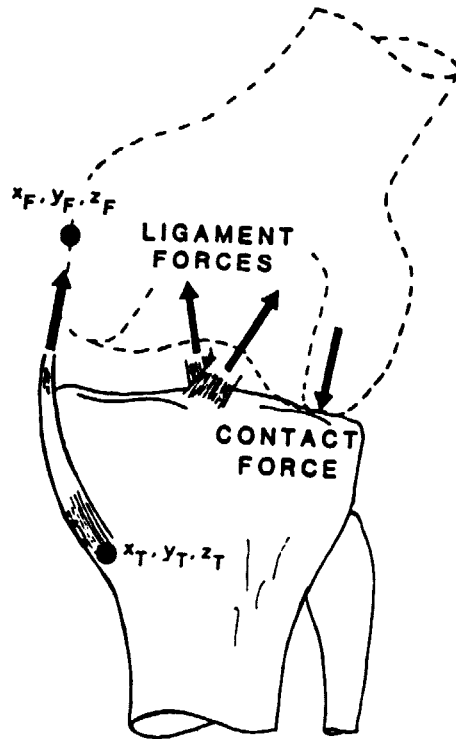
$\{F\}$ = local internal force vector

$[k]$ = local element stiffness matrix

$\{\Delta\}$ = local displacement vector or

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Figure 2-22: Some of the forces acting on the tibia at the knee joint



$$\begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \\ M_{x1} \\ M_{y1} \\ M_{z1} \\ F_{x2} \\ F_{y2} \\ F_{z2} \\ M_{x2} \\ M_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} [k_{11}] & [k_{12}] \\ [k_{21}] & [k_{22}] \end{bmatrix} \begin{bmatrix} \lambda_{x1} \\ \lambda_{y1} \\ \lambda_{z1} \\ \phi_{x1} \\ \phi_{y1} \\ \phi_{z1} \\ \lambda_{x2} \\ \lambda_{y2} \\ \lambda_{z2} \\ \phi_{x2} \\ \phi_{y2} \\ \phi_{z2} \end{bmatrix}$$

$(6 \times 6) \quad (6 \times 6)$

where

λ_{ij} = translational displacement in the i-th direction at segment j

ϕ_{ij} = rotational displacement about the i-th direction at segment j

k_{ij} = stiffness coefficient at i due to a displacement at j or

$$[F_1 \ F_2]^T = [k] [\Delta_1 \ \Delta_2]^T$$

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This equation can be expressed with respect to a global system as:

$$[F_{G1} \ F_{G2}]^T = [k_G] [\Delta_1 \ \Delta_2]^T$$

The transformation between the local and global coordinate system for the stiffness matrix is:

$$[k_G] = [T]^{-1}[k][T]$$

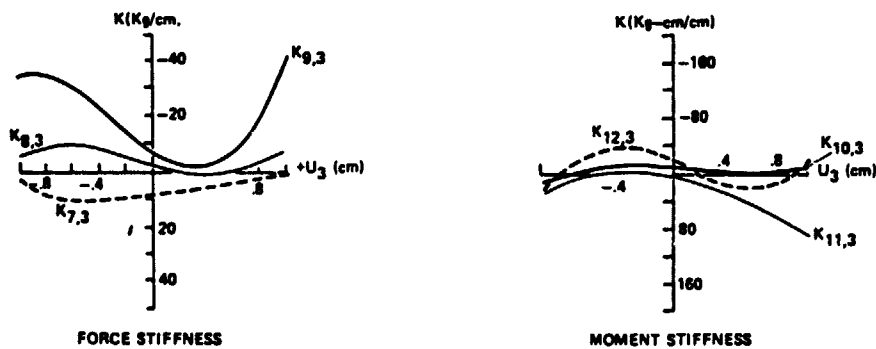
where T is a rotational matrix.

2.11.1. Stiffness Matrix

Since the joint has nonlinear load-displacement characteristics, the stiffness coefficients are not constants, but continuous functions of the applied displacement (rotation) and the initial position of the joint.

Piziali et al. [42] ran a series of test on fresh human knee to determine the stiffness coefficients. Displacements were applied to the femur and the forces transmitted to the tibia were measured. A curve for each k_{ij} as a function of displacement was established (see Fig. 2-23 for examples). Because the stiffness coefficients have only been found for the knee joint, this method cannot be applied to other joints in the body.

Figure 2-23: Stiffness vs. Displacement curves for primary and coupled stiffness resulting from medial and lateral displacement of the femur [42]



Grood and Hefzy [25], analytically calculated the joint stiffness for a given joint position instead of a given joint displacement. The previous stiffness matrix $[k]$ is a secant stiffness matrix. The equation $\{F\} = [k]\{\Delta\}$ can be written in differential form if $\{F\}$ and $\{\Delta\}$ are replaced by their infinitesimal variations $\{dF\}$ and $\{d\Delta\}$ and $[k]$ is replaced by the tangent stiffness matrix $[S]$. The difference between the secant and the tangent stiffness is illustrated in Fig.2-24 The differential load-displacement for the first quadrant of the secant stiffness matrix $[k_{11}]$ is

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$$\begin{Bmatrix} dF \\ dM \end{Bmatrix} = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{Bmatrix} d\lambda \\ d\phi \end{Bmatrix}$$

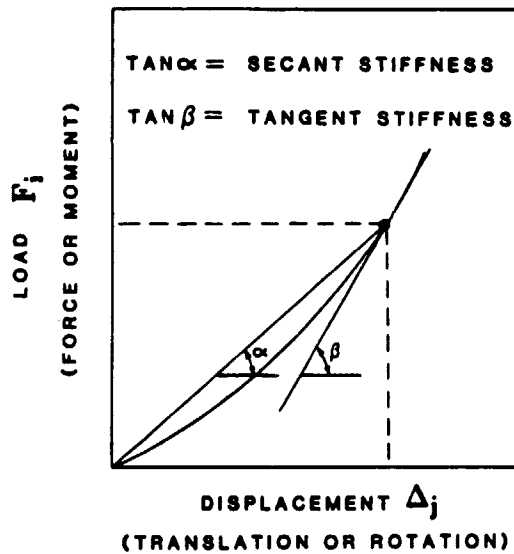
or

$$\begin{Bmatrix} dF \\ dM \end{Bmatrix} = \begin{bmatrix} [\partial F_i / \partial \lambda_j] & [\partial F_i / \partial \phi_j] \\ [\partial M_i / \partial \lambda_j] & [\partial M_i / \partial \phi_j] \end{bmatrix} \begin{Bmatrix} d\lambda \\ d\phi \end{Bmatrix}$$

where the submatrices are 3 x 3. Accordingly, the joint reaction forces and moments due to the ligaments are related to the joint parameter, F, by the integral

$$\{F\} = \int [S] \{d\Delta\}$$

Figure 2-24: Secant and tangent stiffness



The change in joint reaction forces, due to the ligaments, acting on a segment with respect to a translation of an adjacent segment is $[\partial F_i / \partial \lambda_j]$ and was found to be

$$[S_{11}] = \{ (dT/dL) l_i l_j \} + \{ (T/L) \delta_{ij} l_i l_j \}$$

where the relation between tensile force and ligament length was derived from Crowninshield et al. [17] as

$$T = 750A[(L-L_0)/L_0]^2$$

where

L = ligament length

l_i = directional cosines

δ_{ij} = Kronecker-delta

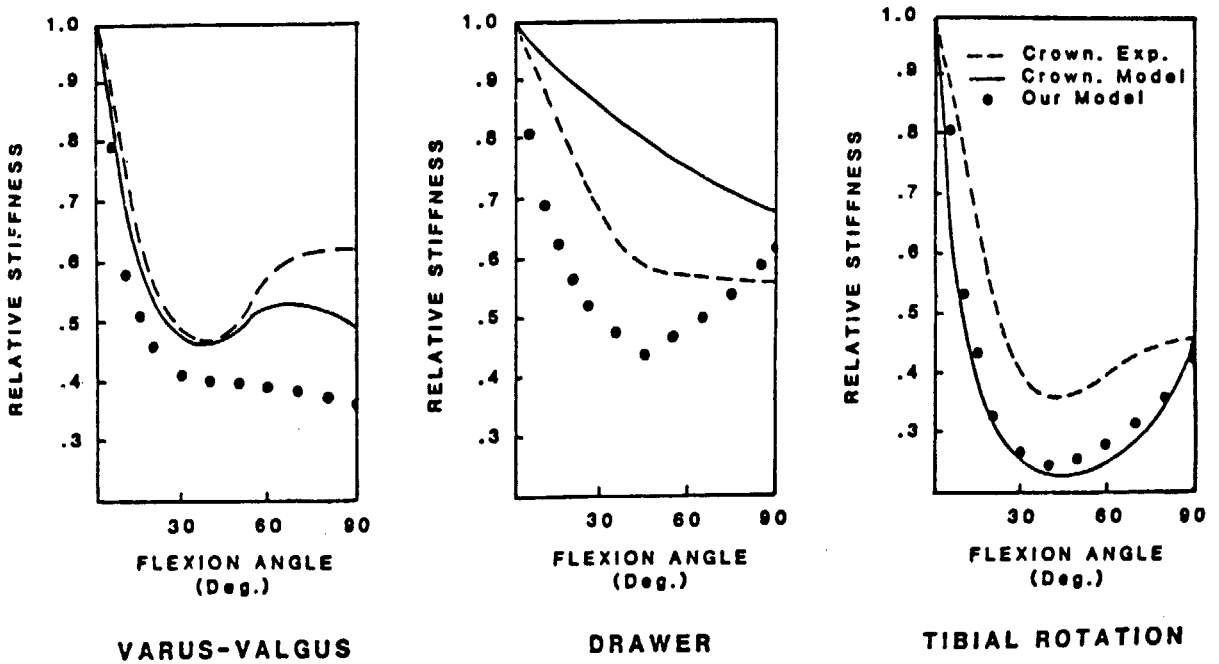
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A = cross sectional area of ligament

L_0 = slack length of ligament

From the equation for $[S_{11}]$, the stiffness is composed of two parts: one due to the ligament's axial stiffness and the other due to a change in the ligament's orientation. Comparisons of predictions of knee joint stiffness with Crowninshield's work are shown in Fig.2-25.

Figure 2-25: Comparison of theoretical and experimental knee stability [25]



2.12. Modeling of Force Distribution

To evaluate the force distribution, the objective function $U = \sum [F_i/A_i]$ with an upper bound constraint of $[F_i/A_i] \leq a$, where $.4 \leq a \leq 1.0$ MN/m² is the simplest to implement in order to get reasonable results. The summation of forces and moments do not provide a reasonable method for determining the maximum force a particular muscle can accommodate. Modeling by the muscle exertion capabilities is physiologically the most desirable but it is shown that muscle exertion capabilities are related to endurance and endurance is related to stress [19]. Nevertheless, An et al. showed that in a static case the objective function of the summation of stresses with a constraint of maximum stress yields similar results to the objective function that is related to endurance. Also, the objective function of the summation

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of stresses can be more easily solved by linear programming than by the nonlinear programming which is required for the endurance objective function.

Ligamentous forces should be considered in the model until it is found that the effect of its forces is minimal. No literature has been found which examines the contribution of ligamentous forces to the strength model. It appears that Grood and Hefzy's modeling of ligamentous forces provides reasonable results without relying on experimental data.

2.12.1. Implementation

The force distribution is considered for computer implementation. Initially a database of body parameters needs to be entered into the system. There may be several databases for the various body types:

1. person of small proportions
2. person of medium proportions
3. person of large proportions

The body parameters for the various body types can be determined either from anatomy books or a sampling of cadavers which fits the required body type. The body parameters are:

1. mass and length of each body segment
2. location of origin and insertion and cross sectional area of the muscles
3. maximum allowable stresses for the muscles
4. cross sectional area and slack length of the ligaments

The variables for a particular problem are the:

1. angles between each body segment to define the body position
2. forces and moments that are applied at the body segments
3. end conditions of the body element, e.g. hand gripping a permanent structure yields zero moment

Once the variables are defined the resultant forces at each joint can be computed by equilibrium equations. The objective function and the maximum allowable stresses provide the equations to use linear programming to calculate the stresses in the muscle system at a particular body position.

2.13. Determining Joint Torque: A Musculo-Skeletal Model

The optimization model is a backward analysis of strength. Given a joint torque, it determines the distribution of force among the muscles. Alternatively, the joint torque can be determined from a given set of muscle forces. In this direct analysis of strength, the muscle

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forces are dependent on the physiological makeup of the muscles. That is, they are derived from the force capabilities of the muscles. Once these forces are known, the resultant torque is set by the position of the muscles, or more precisely, by the direction of the forces with respect to the limbs.

2.13.1. Hill's Equation of Muscular Behavior

Many studies have been made on the characteristics of isolated muscle. Most notable are those carried out by Hill [26]. From experiments on the heat of shortening in frog muscles, he derived a hyperbolic relationship between contractile force, P , and the velocity of contraction, V :

$$(P+a)(V+b) = b(P_o+a)$$

where P_o is the maximum isometric strength at rest length. The parameters a and b are proportional to the muscle's cross-sectional area and length, respectively. Moreover, Hill found that $a/P_o = b/V_{\max} = \text{constant}$. The interpretation was that force is also proportional to muscle area and length.

2.13.2. Parameter Values

Although the values of a and b vary from muscle to muscle, Hill claims that $a/P_o = b/V_{\max} = \text{constant}$, where V_{\max} is the maximum velocity of contraction at rest length. He obtained values of 0.2 to 0.5 for the constant. Ralston [44], however, obtained a value of 0.81 for the human pectoralis major, and Fenn and Marsh [22] obtained a value of 0.75 for cat quadriceps. The disagreement on this value has been attributed to changes in the dimensions of the muscle as it contracts. Several studies [1] [33] have suggested that using instantaneous isometric strength instead of the isometric strength at rest length would account for these changes. Accordingly a/P_{o1} , not a/P_o , is constant over all muscles. With $a/P_{o1} = b/V_{\max} = c$, Hill's equation becomes

$$P = P_{o1} \left[1 + \frac{(1+c)V}{(b+V)} \right]$$

To obtain P_{o1} , Pedotti et al [40] linearized the muscle tension-length curve for small changes in length around the rest length:

$$P_{o1} = P_o [1 - 1.25(l_o - l)] \quad \text{for} \quad l < l_o$$

$$P_{o1} = P_o [1 - 0.5(l - l_o)] \quad \text{for} \quad l > l_o$$

For greater changes in length, Stern [49] obtained descriptive equations for the bicep muscles by fitting experimental data :

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$$\frac{P_d}{P_o} = 100 - \frac{[\log_{10}(l/l_o)]^2}{0.054448} \quad \text{for} \quad P_d/P_o < 1.078$$

$$\frac{P_d}{P_o} = 2.1277 - 0.010638(l/l_o) \quad \text{for} \quad 1.078 < P_d/P_o < 2.00$$

The values of P_o and V_{max} can be obtained by:

$$P_o = k(\text{cross-sectional area})$$

where k = force per unit area of muscle.

$$V_{max} = [\text{mean fiber length}][R_F G_F + R_S G_S]$$

where R_F is the percentage content of F-type fibers in the muscle and G_F is the speed characteristic of F-type fibers; R_S and G_S are analogous for S-type fibers.

There is a wide range of values for k . Pedotti [40] gives a value of $k = 15 \text{ kg/cm}^2$ for the locomotor muscles. Bourne, however, reports values of k ranging from 2.4 kg/cm^2 to 9 kg/cm^2 for the quadriceps femoris, and Haxton reports 3.9 kg/cm^2 for the calf muscles. Ralston obtained 2.4 kg/cm^2 – 4 kg/cm^2 for the biceps brachii; however, using ultrasonics, Ikai and Fukunaga obtained a value of 9 kg/cm^2 (as compiled by Bourne [9]). The variance may be partially due to different fiber arrangements: parallel and pennate. In pennate arrangement, the angle of the fibers with respect to the length of the muscle, along with cross-sectional area, determines the power of the muscle. Moreover, the different types of muscle fibers differ in their diameter and peak tension. F-type fibers have larger diameters and can achieve twice as much tension as S-type fibers.

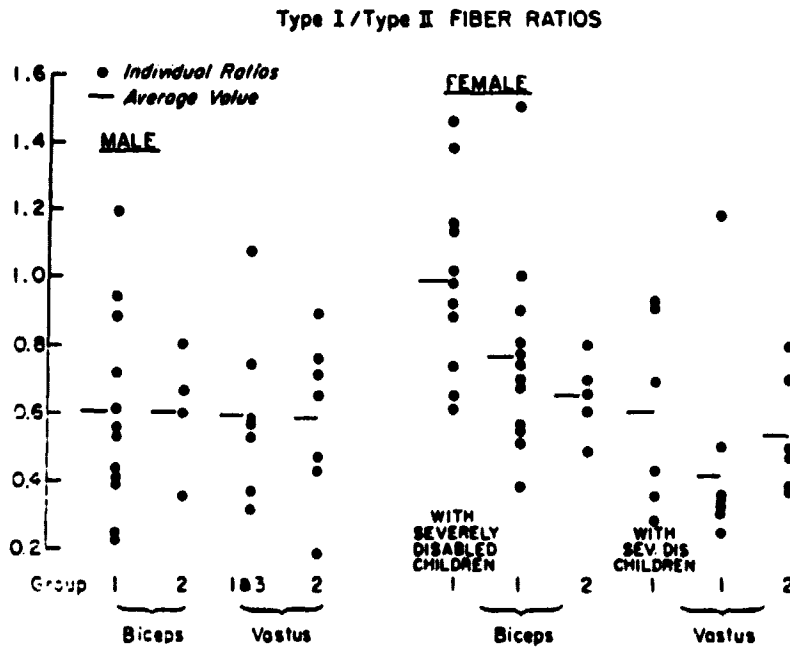
Although these characteristics are well known, there is very little data on actual muscle content. One study [10] took muscle biopsies to determine fiber content of the biceps and vastus muscles. The ratio of type I to type II fibers are shown in Table 2-26. For the men, the average of the ratios was constant at 0.6, but for the women, it varied from 1.0 to about 0.5 (Fig. 2-26)

2.13.3. Computer Implementation

The musculo-skeletal model integrates the muscular system and the skeletal system to determine the torque at a joint. Maximum stimulation is assumed so the nervous system can be ignored. Each muscle is represented by its directed line of force (Fig. 2-27) This model was implemented as a computerized, iterative process. The Fortran listings can be found in Appendix II. It requires as input physiological parameters describing the muscle(s) of interest

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Figure 2-26: Ratio of type I to type II fiber found by muscle biopsies



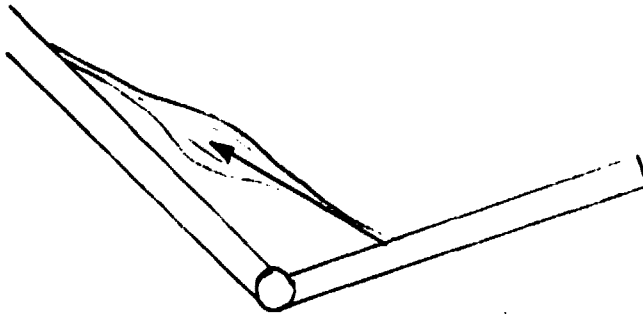
and the placement of the muscle(s) in the body:

1. rest length
2. insertion length
3. origin length force per unit area
4. cross sectional area
5. value of Hill's constant
6. velocity of contraction
7. range of motion

For each moment of time, the model uses Hill's equation to derive muscular force. Then from the body position, calculates the corresponding torque.

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Figure 2-27: The musculo-skeletal model



2.13.4. Results

The model was run for flexion of the biceps brachii. This muscle was used for the test mainly because there is more available data for the biceps. Physiological values for a hypothetical biceps muscle were compiled from Stern [49], Fick [23], and Schumacher [45]:

- length of origin = 3.2 cm
- length of insertion = 27.0 cm
- cross-sectional area = 3.7 cm²
- force per unit area = 10.00 N/cm²
- maximum velocity = 25 rad/s

The value c was taken arbitrarily at 0.5, and velocity of contraction at 24 rad/s. The rest length was taken to be the length at 90 degrees of flexion. The model was also run at different values, and the results compared to the results of these reference values.

Graph 1 compares the results in using Hill's equation with P_0 and with P_{01} . For this muscle, at least, there is very little difference in the calculated torque. The remaining tests were done using Hill's equation with P_{01} . Graph 2 shows the difference in internal muscle force and external torque. Muscle force remained essentially the same throughout flexion, but external torque ranged from about 0.0 to 84.04 N-cm. In Graph 3, the constant c was varied from 0.2 to 0.8 while all other parameters remained the same. In Graph 3a, the curve of $c=0.4$

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was taken to be the normal, and the external torque relative to the norm was calculated for each time period. For each change in c , external torque was multiplied by a constant factor. In other words, change in external torque is proportional to the change in c . In Graph 4, the force per area was varied from 10 to 20 N/cm². Graph 4a is a normalized graph using the curve of $f/A = 10$ as the normal. It can be seen that the change in external torque is also proportional to the change in force per unit area. Graph 5 shows the changes in external torque due to changes in cross-sectional area. Graph 5a is a normalized graph with $A=3.7$ as the normal. Change in torque is also proportional to change in area. Graph 6 shows the changes due to changes in velocity of contraction. Normalization was done by taking the ratio of the torque at each time period relative to the peak torque for each curve. The results are shown in Graph 6a. Graphs 6 and 6a show that at higher speeds, the peak torque and the change in torque is greater. (see Appendix III for graphs)

2.13.5. Discussion

The musculo-skeletal model is based on an integrated profile of muscle physiology and biomechanics. The nervous system also determines strength by controlling the amount and frequency of stimulation. However, for this model, stimulation was taken to be at maximum so that the nervous system can be eliminated from the factors. The model, then, determines the maximum strength for a given muscle and its placement in the body. Since contractile force is linearly related to nervous stimulation, for any other level of stimulation, strength can be taken as the corresponding percentage of the maximum.

As described in the introduction, motivation can also influence the amount of strength that is actually exerted. Because motivation is psychological, its effects are difficult to quantify. However, it can also be integrated into the model as an offset factor whose value can be taken to fit experimental data:

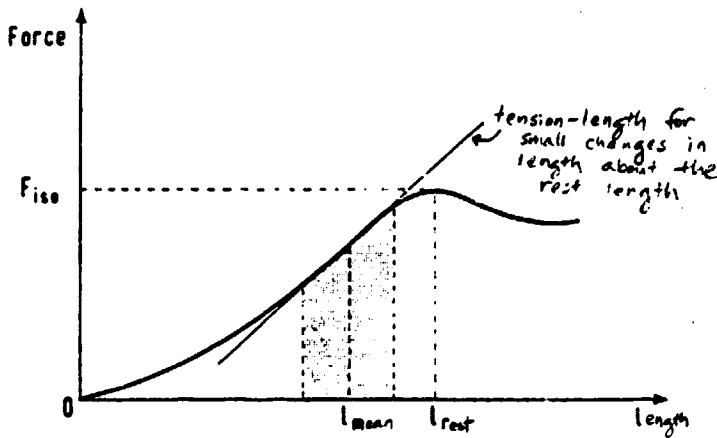
$$\text{actual strength} = r(\text{calculated strength})$$

where $r \leq 1.00$.

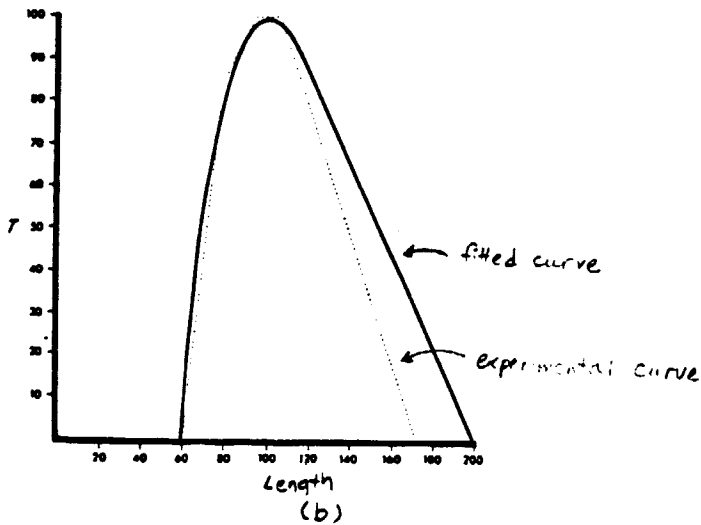
The model is dependent on the instantaneous isometric strength. Both Pedotti [40] and Stern [49] fitted experimental data to obtain the isometric strength at lengths other than rest length. Pedotti's equation is valid only for very small changes about the rest length. The curve is similar to the experimental isometric force-length curve of biceps muscles obtained by Ralston [44]. Preliminary tests, however, showed that for the composite biceps muscle, the changes in length during flexion were greater than the changes allowed in Pedotti's equation. Stern's equation allowed greater changes. However, the curve described by the equation is not consistent with Ralston's curve (Fig. 2-28).

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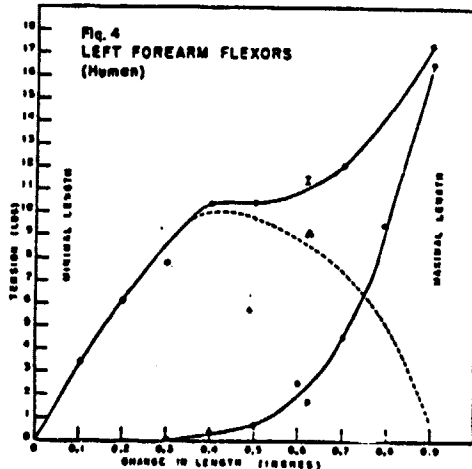
Figure 2-28: Comparison of the Pedotti equation and the Stern equation for the muscle tension-length relationship



(a)
Pedotti's equation



Stern's equation



(c)

Ralston's experimental data for human biceps

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The model relies on many other physiological values: the length of insertion, the length of origin, the cross-sectional area, etc. Because of the lack of consistent data, the model is difficult to evaluate. Where data is available, there is often disagreement on the actual values, such as on the value of the force per unit muscle area. The biceps muscle used for the tests was put together from various sources. It may be that this composite profile is not realistic, but that were the only available data. The value of the constant c has been reported to range from 0.8 to 0.25. For the test, it was taken arbitrarily at 0.5. There is no basis for taken at this value other than that it is within the reported range.

2.14. Summary of Force Calculations

This report investigates two area of strength modeling at the torque level:

1. inverse calculation: to find the reactive forces for a body when it is applying or subjected to external forces
2. calculate forces: to find the force that a body is capable of exerting from a certain restrained position

Inverse calculations can be answered directly by equilibrium equations when the body and the end conditions render the force system determinant. By using coordinate transformations the forces at the joint level can be found. Optimization methods, which approximate the force/stress distribution, can be used to determine whether the muscles can hold the body in a particular quasi-static position. Also the analytical method to find the contribution of the ligaments can be introduced into the force structure. The optimization method relies on various physiological values: the point of origin, the point of insertion and the cross sectional area of the muscles. Calculations of the ligament forces are subject to ligament length, ligament's slack length, and cross sectional area. Experimental data on these values deviates widely and, therefore, approximations would have to be assumed.

Hill's equation of muscle behavior directly solves for the force capabilities at a joint (torque). Applying Hill's equations at various joints the force capabilities of a body at a specified position can be found. Transforming the joint forces (local coordinates) to the point where the body is applying the external forces (global coordinates) yields the force capability of a body that is in a particular position. Hill's equation is dependent on experimental data for instantaneous isometric strength, and other physiological values which requires much more experimental data.

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3. Human Strength Databases

There are several strength databases (e.g. [5, 6]) which would be more useful if made available through computer database query systems. The data returned by the query would typically be interpolated to provide approximate strength data for positions not measured directly.

Much of the data available on human body strength is largely tabular in format [5, 6] but there seems to be little consistency between the various tables. Also, while there is quite a variety of data, it is rather fragmented. For example, there might be two tables of strength data presented as the maximal force applied for a given set of parameters. This method of presentation seems to be common. It is apparant from these examples that for two different measurements there may be a great difference in the way the data is collected and packaged.

The problem of storing strength data in a database does not appear to be a difficult one. The database model most appropriate to the problem is the *relational* one. In this model, tables of data can most conveniently be represented as relations with similar or functionally associated tables combined to form single relations. Storing each table or associated group of tables as a separate relation offers a solution to the problem caused by the variety of formats encountered. A relation can be constructed to contain all the pertinent information contained in a given table of data. The majority of the attributes of such a relation would be key information and perhaps some statistical information of potential use in more complicated queries. Information about each table as a whole can be included as accompanying documentation. Here the assumption is made that the person programming the application which will use the database will be knowledgeable with regard to its design and content.

As an example, the relation created to store the information contained in a table of "maximal static hand forces at various elbow angles exerted on a vertical handgrip by seated males" (Table 2.5-4 of [50]) might have the following attributes: HAND, which specifies the hand being used; DIRECTION, which indicates the direction of the force being exerted; ANGLE, indicating the elbow angle; and FORCE, which would contain the actual data. In addition, there might be included some attributes for the percentiles and standard deviation. The attributes HAND, DIRECTION, and ANGLE would comprise the key for this relation. Here it would be desirable, but not imperative, to have attributes such as HAND and DIRECTION declared over enumerated data types. A main difference between the various tables in the literature is the number of key attributes needed to identify the desired datum.

The next step in the development of the database is the design of a set of queries to access the relations containing the strength data. This step will require some input from the potential

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users of the system. Before designing the queries it will be necessary to know what strength information is required and in what form. It may not be desirable to integrate such a database completely into TEMPUS unless the data collected were complete and extensive enough to satisfy a large class of potential strength information requests. The alternative is to use the strength database as a separate adjunct to the OSDS software environment which is called upon when required.

Note that there is nothing in the database approach which precludes incorporating strength information on a particular individual. Just as TEMPUS permits the use of specific individuals as well as anthropometrically generic (statistically derived) bodies, so too could databases have data for both types. The advantage to the relational structure of the database is clear: the type of individual (real/generic) is just another table attribute.

There are two problems with the database approach, though neither is insurmountable: converting tabular data to computer readable files, and interpolating over arbitrary body position and orientation. The first problem may be solved by determining which research projects produced computer readable data and obtaining such data on suitable magnetic media. It may be faster, however, to simply resort to manually entering the desired data. The second problem is the more severe. The data that is available may not be taken over enough variables to permit the safe interpolation of strength values at in-between positions and orientations. The missing information may, however, be collected through specific experiments designed to complete the database. For example, consider the strength data published by NASA [50]. Figures 2.5-2, -3, and -4 of that report are graphs for "predicted equal hand force capabilities" for both shirtsleeved and suited individuals. The graphs show three force directions (lifting, pushing, and pulling) for low (0.2) gravity situations. The data is provided in graphical form and would need to be obtained in the original numerical form *prior* to graphical contour analysis, presumably available from the original source [12]. Moreover, the data is only valid for "horizontal hand position in front of ankles." For data on other hand orientations, arm positions, or zero-gravity one would have to resort to further experimentation.

The interpolation problem is interesting, but simple solutions may suffice. For example, if data is required on arm strength when the upper arm is at a particular angle ([halfplane, deviation, twist] in TEMPUS parlance), the values from the database at the three closest angles may be used to linearly interpolate a solution. In essence, the three known strength values determine a surface over the sphere of motion of the arm. Data points on the interior of the triangle of known data points may be interpolated by weighting each known data value by the distance from the known data point. (By choosing three points we are more likely to cover a non-trivial area of the possible motion sphere of the shoulder.) The possibility of having a twist

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value that is not in the database at all, however, leaves this "pure" interpolation technique open to substantial errors. If this is expected to be a frequently needed computation, then it will be worth making a series of detailed strength measurements to adequately cover the spherical surface within the joint limits of the shoulder. Linear interpolation will become quite satisfactory as the number of known data values increases. By collecting such data systematically over several fixed twist values, a complete and accurate strength map could be generated. Moreover, this map would be directly keyed to the TEMPUS parameters already describing limb position and orientation.

The direction recommended here is to study the available strength databases and build, in as uniform a fashion as possible, a relational database storing tabular data. For this purpose we have examined such databases and have obtained RDB, a relational database product from Digital Equipment Corporation for the VAX computer. We are presently attempting to place several relevant strength databases into RDB format and then use the RDB query system to obtain numerical information for subsequent interpolation.

4. Data Acquisition Methods

There are direct and indirect methods for measuring strength data. The direct methods output forces; the indirect methods output a sequence of (joint) positions over time which may be converted to velocities and accelerations, and then to forces if masses and moments of inertia are known. Direct methods connect a body part to a suitable sensor to measure force, for example, a forceplate or a Cybex sensor. The indirect methods use mostly passive (non-connected) sensor systems to determine joint position, for example, Selspot scanners and image analysis (digitization). There are also direct position sensing devices such as three-dimensional sonic digitizers and the six-dimensional electromagnetic technology sensor used in the Polhemus digitizer.

Direct force sensors generate force information which may be fed into one or more of the graphical display methods described in Section 5.

Indirect force (position) sensors must have their outputs processed to produce smooth data curves over time. Without data filtering, the computation of accelerations from changing positions, for example, is extremely sensitive to noise and even numerical errors in the (finite resolution) input data. Techniques such as Fourier analysis and filtering, or simple geometric curve smoothing are used to control the unwanted variability.

The problem of collecting and analyzing motion data from which strength may be assessed is discussed further in the Motion Analysis report [7].

5. Graphical Display of Strength Data

In its simplest form, strength data is a collection of parameters describing certain force or torque capabilities of some human body unit. Viewed in the abstract, the display of these parameters could utilize methods for any multi-parameter system. There are several such methods which will be reviewed briefly below. Then we will examine more specific graphical techniques that attempt to convey the meaning of a parameter as well as its value.

There have been several attempts to graphically present multi-dimensional data [51]. Most methods depend upon the astute understanding of the problem domain by the graphical designer. A few general methods are known, but all suffer from various defects. The most important limitation to understanding multi-dimensional data lies in the human information processing ability to perceive and compare distinct multi-dimensional features within some global presentation. Thus one finds methods such as *sine functions* [4], *Chernoff faces* [14], *hypergraphics* [28], *sound* [55], and *multi-sensory presentations* [8].

Unfortunately, these methods, by their very generality, are not as visually effective in presenting the *human body-specific* semantics of strength parameters. That is, these general techniques for parameter display do not tie the parameter to the body property it describes.

To solve this problem we must develop methods to use the body itself as a context for the parameter display. The key features available on the body itself, when rendered graphically are:

- The body segment chains of interest may be highlighted,
- The solid body surface may be intensity or color coded.
- The direction of motion of a body joint may be indicated with, *e.g.*, an arrow.
- The orientation of a body joint or segment may be indicated with a coordinate axis *gnomon* pointing in the three principal coordinate directions.
- The reachable space of a body chain may be displayed as a polyhedral volume (as computed by Jim Korein's workspace algorithm described in his PhD dissertation).
- Single or multi-dimensional parameters may be displayed with standard graphing techniques (bars, disks, pies, graphs, *etc.*) at specific body points (joints), such as the reactive forces where the body contacts an environmental object.
- Comparative values across two or more individuals may be displayed in adjacent viewports on the graphics display.
- Area deformation techniques, such as varying the size of a segment according to some strength parameter, can also be used for comparative purposes.
- Temporal sequences of parameter values for one or more individuals may be

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displayed over time through an animation.

- Real-time motion dynamics may demonstrate particular behaviors.

Although this list is certainly not exhaustive, it does provide hope that the multi-dimensional nature of strength may be portrayed visually and symbolically. Given the outline of strength parameters in the introduction to this report, there are several possible mappings of parameters onto the possibilities above. For example:

- For a fixed body segment (say, the arm) display a pencil of vectors at the wrist whose length or color is proportional to the nominal (static) force exorable in the vector's direction. In the limit, the color coding could be applied to a fixed size sphere centered at the wrist: the color or intensity of each point on the sphere would correspond to the strength in that direction. By drawing such vector pencils or colored spheres at suitably spaced locations about the reachable space, a more global view of strength distribution and variation would be visible. This method will also work to some extent for effector positions inside the extremes of the reachable space. The spheres will simply appear "inside" and could be examined more closely by suitable graphical viewing operations.
- Draw the reachable space of the arm, say, as a polyhedron ("workspace" in TEMPUS). For each vertex of the space encode a single parameter, for example, the maximum exorable force, as an intensity or color. Use vertex-to-vertex interpolation in the visible surface graphics rendering to shade the polyhedron with interpolated colors (strengths).
- Given an applied force or forces on the body (with or without external constraints), encode the maximum torque at each body joint in a color. The "BUBBLEpeople" models could be especially effective if the spheres at the joints were assigned the indicator colors. Reaction forces could be displayed in a similar color scale, but at the points of contact on the environmental objects, to distinguish them from the applied forces.
- Display color changes to the above models to demonstrate the effects of fatigue on any of these parameters. By using the raster display's color table, the changes could be shown in actual (real-time) or compressed time without display redrawing delays.
- Since motion dynamics will be displayable through TAN, forces may be drawn in graph form for particular limb masses. Several of these could be overlaid to illustrate the effects of body size, fatigue, effector position, etc.

Surely other techniques and variations of these suggestions will be possible.

6. Conclusions

The basic conclusions reached in this strength modeling report are that

- Strength models based on muscle action are complex but may be implemented,
- A full scale kinematics and dynamics model of the body is necessary to properly handle arbitrary restraints and external forces,
- Strength databases ought to be brought on-line to satisfy some standardized

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strength queries,

- The graphical display of empirical or computed strength information is feasible and desirable.

To achieve these goals, parallel efforts may be mounted. The first two items are related in the sense that adequate muscle models will provide data for the complete kinematics and dynamics simulation of the body. The database will also provide some of the necessary information, but can also be used independently. The graphical display methods can, of course, be applied to any strength data or modeling technique.

The database should be constructed starting with empirical data on upper arm and torso strength. The relations necessary must be elaborated and the database entered. Next, the interpolation methods must be designed and coded. At this point, graphical display tools will become useful in providing visual feedback from database queries. This database system may be kept independent of TEMPUS, though it should clearly retain as much commonality (*e.g.* naming conventions, joint angle specification) with the TEMPUS body structure as possible. After proper evaluation of the database and consultation with potential users, the database should be expanded to include other body strength data.

The muscle models should be implemented to give a simulation of an isolated joint or limb. The user must be allowed to specify the values (or defaults) of parameters which control various aspects of the muscle force equations. Experience with this model should lead to an evaluation of the potential of the muscle model for accuracy in the tasks expected. If the evaluation is satisfactory, the model should be extended to a complete kinematics and dynamics simulation of the body. This task will require the integration of a general mechanical problem solver into TEMPUS such that the body model positions and applied forces are transformed into the format required by the simulation. During this phase, TEMPUS must be extended to permit a user to specify external forces acting on a body. By definition, these forces include arbitrary restraints on any part of the body. Necessary parameters for full body dynamics must be determined, possibly through experiments at the AML. The simulation model must be thoroughly tested and refined as needed to ensure a valid dynamics model.

Concurrently, the graphical display of strength data must continue to be developed, including the visual correlation of strength data with regions of the body and the real-time display of forces (restraints) and the body's reactions to them.

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7. Schedule and Resources

The tasks outlined in the Conclusion could be realized over a three year period if suitable personnel were directed to its implementation. The schedule would, of course, differ if other directions were taken. In particular, the construction of a relational database of existing strength data would take only a year, while the more complete muscle and dynamics models will take longer to build, collect parameter values, and validate. If much data is needed which is not available, then validation and testing could extend beyond the three year period. The approximate timetable for a human strength modeling system is given in Table 7-1.

Table 7-1: Strength Modeling Schedule

Time Milestone	Task (per staff member)
year 0.5	Build relational database of upper arm and torso strength. Build simple (isolated) muscle model and test; evaluate existing kinematics/dynamics solution systems.
year 1	Develop interpolation methods for strength database. Interactive graphics specification of restraints and graphical display of strength data from database.
year 2	Extend database to other body areas and obtain new data from JSC AML. Integrate TEMPUS body with dynamics.
year 2.5	Determine parameters for full body dynamics. Body correlated graphical display of strength data.
year 3	Test and refine full body kinematics/dynamics simulation. Validate full body dynamics model against empirical data.

The *time milestone* is the length of time from project inception (not a duration) to the completion of the indicated *tasks*. The tasks are a summary of the work needed to fulfill the system requirements discussed in the Conclusion. Each task refers to *one* graduate research assistant. This is a half time load (20 hours/week). Thus multiple tasks for one time milestone are assumed to proceed in parallel, and a total of two individuals for three years are required.

The resources required are summarized in Table 7-2. The monetary estimates are based on solely on 1985 University of Pennsylvania rates including employee benefits, tuition, and overhead as applicable. There is no provision for inflation; that may be projected by NASA as necessary.

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Table 7-2: Strength Modeling Resources

2 Graduate Research Assistants for duration of project.....\$50K/year
Faculty supervision time (10% of academic year).....10K/year
Equipment:
RDB (relational database for VAX).....6K
Travel, current expense, duplicating, etc.....34K/year

Totals:

Year 1: \$100K (includes RDB)

Year 2: \$ 94K

Year 3: \$ 94K

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Strength Modeling

I. Force Analysis Programs

```
c      - general program for analysis of forces and moments of a n-link
c      system that can be used with any homogeneous transform.
c      - can compute the forces for either forward or backward i number of
c      links
c
c      read in n-links
c      read in parameters for each i-th link: alpha & adistance
c      read in applied local forces and moment at j-th coordinate frame
c      [note can use superposition to get the result of several forces]
c      read in initial positions for each i-th link: theta & distance
c      if transforming to the i-th minus 1 frame:
c           $F(i-1) = \text{inv}[A(i)] * F(i)$ 
c      if transforming to the i-th plus 1 frame:
c           $F(i+1) = A(i+1) * F(i)$ 
c      options: new position, transforming forward, transforming backward
c
c      if want to interactively change position variables - requires 2 files:
c          - parameter file: alpha,adistance
c          - variable file : theta,distance,no.
c
c      change position:
c          - relative and absolute position
c
c
c      PLOK.FOR
c
c      real AT(4,4), AT_cur(4,4), AT_pre(4,4)
c      real alpha(20), adistance(20), theta(20), distance(20)
c      real alpha_no, adistance_no, theta_no, distance_no
c      real force(3), moment(3)
c      real PI,RADIAN
c      integer no_links, node_cur, node
c
c      write(6,*)'READ IN NUMBER OF LINKS - limit to 20'           !parameter file
c      read(5,*)no_links
c
c      write(6,*)'READ IN ALPHA AND ADISTANCE'
c      do i=1,no_links
c          read(5,fmt='(2f10.4)')alpha(i),adistance(i)
c      enddo
c
c      write(6,*)'READ IN APPLIED FORCE AND MOMENT'                 !assume 1 force
c      read(5,fmt='(3f10.4)')(force(i),i=1,3)                     !3 space
c      read(5,fmt='(3f10.4)')(moment(i),i=1,3)
c
c      write(6,*)'READ INITIAL VARIABLE PARAMETERS'               !variable file
c      do i=1,no_links
c          read(5,fmt='(2f10.4)')theta(i),distance(i)
c      enddo
c
c      BACKWARD TRANSFORM - TO ORIGIN
```

Strength Modeling

```
node_cur = no_links

call matidentity(AT_pre,4)
PI=3.14159
RADIAN=PI/180.0
do while (node_cur .ne. 0)
    alpha_no = alpha(node_cur)*RADIAN
    theta_no = theta(node_cur)*RADIAN
    adistance_no = adistance(node_cur)
    distance_no = distance(node_cur)

    call atransform(AT_cur,alpha_no,adistance_no,theta_no,distance_no)

    write(6,*) 'A TRANSFORM'
    do i=1,4
        write(6,fmt='(4f10.4)')(AT_cur(i,j),j=1,4)
    enddo

    call ainverse(AT_cur)

    write(6,*) 'INVERSE A TRANSFORM'
    do i=1,4
        write(6,fmt='(4f10.4)')(AT_cur(i,j),j=1,4)
    enddo

    call matply(AT_cur,AT_pre,AT,4,4,4)

    write(6,*) 'A TRANSFORM - AFTER MATPLY , BEFORE STATIC'
    do i=1,4
        write(6,fmt='(4f10.4)')(AT(i,j),j=1,4)
    enddo

    call static(AT,force,moment)
    call matcopy(AT,AT_pre,4,4)
    node_cur = node_cur-1
enddo

end
```

Strength Modeling

```
c      'A' transform matrix for a revolute joint
      subroutine atransform(A,alpha,ad,theta,d)
      real A(4,4), theta, alpha, ad, d
      real cth, sth, cal, sal
```

```
      do i=1,4
        do j=1,4
          A(i,j)=0.0
        enddo
      enddo
      cth=cos(theta)
      sth=sin(theta)
      cal=cos(alpha)
      sal=sin(alpha)
      A(1,1)=cth
      A(1,2)=-sth*cal
      A(1,3)=sth*sal
      A(1,4)=ad*cth
      A(2,1)=sth
      A(2,2)=cth*cal
      A(2,3)=-cth*sal
      A(2,4)=ad*sth
      A(3,2)=sal
      A(3,3)=cal
      A(3,4)=d
      A(4,4)=1.0
      return
      end
```

```
c      ainverse.for
c      inverse of 'A' matrix
```

```
      subroutine ainverse(AT)
      real AT(4,4)
      real n(3), o(3), a(3), p(3), adumb
      do i=1,3
        n(i)=AT(i,1)
        o(i)=AT(i,2)
        a(i)=AT(i,3)
        p(i)=AT(i,4)
      enddo
      do i=1,3
        AT(1,i)=n(i)
        AT(2,i)=o(i)
        AT(3,i)=a(i)
      enddo
      call dotprd(p,n,adumb)
      AT(1,4)=-adumb
      call dotprd(p,o,adumb)
      AT(2,4)=-adumb
      call dotprd(p,a,adumb)
      AT(3,4)=-adumb
      AT(4,4)=1.0

      return
      end
```

Strength Modeling

```
c      matply.for
c      multiply matrices
c      A[row,ndim] x B[ndim,bcol] = C[arow,bcol]

      subroutine matply (a,b,c,iarow,jbcol,ndim)

      real a(4,4), b(4,4), c(4,4)

      do i=1,ndim
        do j=1,jbcol
          c(i,j)=0.0
        enddo
      enddo

      do i=1,iarow
        do j=1,jbcol
          sum = 0.0
          do k=1,ndim
            sum=sum+a(i,k)*b(k,j)
          enddo
          c(i,j)=sum
        enddo
      enddo

      return
      end

      subroutine matidentity(mat,n)
      real mat(4,4)
      integer n

      do i=1,n
        do j=1,n
          mat(i,j)=0.0
        enddo
        mat(i,i)=1.0
      enddo

      return
      end

      subroutine matcopy(old,new,irow,icol)
      real old(4,4),new(4,4)
      integer irow,icol

      do i=1,irow
        do j=1,icol
          new(i,j)=old(i,j)
        enddo
      enddo

      return
      end
```

Strength Modeling

```
subroutine static(AT,force,moment,node_cur)
```

```
real AT(4,4)
```

```
real force(3), moment(3), force_A(3), moment_A(3)
```

```
real garb(3), garbage(3), n(3), o(3), a(3), p(3)
```

```
integer i,j,m, node_cur
```

```
c write(6,*) 'STATIC FORCE ANALYSIS BY VIRTUAL WORK'
```

```
c write(6,*) ' A TRANSFORM'
```

```
c do i=1,4
```

```
c write(6,fmt='(4f10.4)')(AT(i,j),j=1,4)
```

```
c enddo
```

```
write(6,*) ' APPLIED FORCES '
```

```
write(6,20)(force(i),i=1,3)
```

```
write(6,*) ' APPLIED MOMENTS '
```

```
write(6,20)(moment(i),i=1,3)
```

```
do i=1,3
```

```
    n(i)=AT(i,1)
```

```
    o(i)=AT(i,2)
```

```
    a(i)=AT(i,3)
```

```
    p(i)=AT(i,4)
```

```
enddo
```

```
call crossprd(force,p,garb)
```

```
write(6,51)(garb(i),i=1,3)
```

```
51 format(' f x p = ', 3(1x,f10.4))
```

```
call vectop(garb,moment,garbage,1)
```

```
write(6,52)(garbage(i),i=1,3)
```

```
52 format(' f x p + m = ', 3(1x,f10.4))
```

```
call dotprd(n,force,force_A(1))
```

```
call dotprd(o,force,force_A(2))
```

```
call dotprd(a,force,force_A(3))
```

```
call dotprd(n,garbage,moment_A(1))
```

```
call dotprd(o,garbage,moment_A(2))
```

```
call dotprd(a,garbage,moment_A(3))
```

```
write(6,*) ' FORCES IN THE A COORDINATE FRAME'
```

```
write(6,20)(force_A(i),i=1,3)
```

```
write(6,*) ' MOMENTS IN THE A COORDINATE FRAME'
```

```
write(6,20)(moment_A(i),i=1,3)
```

```
20 format(3f10.4)
```

```
return
```

```
end
```

Strength Modeling

```
subroutine crossprd(a,b,c)
real a(3), b(3), c(3)
do i=1,3
    c(i)=0.0
enddo
c(1) = a(2)*b(3) - b(2)*a(3)
c(2) = a(3)*b(1) - a(1)*b(3)
c(3) = a(1)*b(2) - a(2)*b(1)
return
end
```

```
c    dotprd.for
c    dot product
subroutine dotprd(a,b,c)
real a(3), b(3), c
c= 0.0
do i=1,3
    c=a(i)*b(i)+c
enddo
return
end
```

```
c    vectop.for
c    vector operations for 3d space
c    (n=1) addition
c    (n=2) subtraction
subroutine vectop(a,b,c,n)
real a(3), b(3), c(3)

do i=1,3
    c(i) = 0.0
enddo
do i=1,3
    if (n .eq. 1) c(i) = a(i)+b(i) !add
    if (n .eq. 2) c(i) = a(i)-b(i) !sub
enddo
return
end
```

Strength Modeling

II. Strength Programs

```

program isokinetic

character*75 fnam, mus
character ext
real rest_len, insert, orig, cross, funit, vmax
real constc, starta, enda, full, vel, tim, torq, iso
common /fbl/ fnam, mus, ext
common /sbl/ rest_len, insert, orig, cross, funit, vmax
common /thbl/ constc, starta, enda, full, vel, tim, torq, iso

real len, inst_p, forin, ang, tang
real interv
logical check

c   { program to obtain isokinetic strength }

c   { input parameter values }
100  continue
    call inparam

c   { check values }
    call wrchk(check)
    if (.not. check) go to 100

c   { values okay - open file and calculate values }
    open (unit=5, file=fnam, status='new', carriagecontrol='list')
    call dtor(starta, enda, full)
c   { interv is the change of angle per unit time }
    interv = vel*tim
    ang = starta
c   { write parameter values to file }
    call wrstat
c   { write header for calculated values }
    call wrlab
c   { use iterative process to calculate force and }
c   {   torque for each time period               }
    do while (ang .le. enda)
c       { determine angle between limbs }
        if (ext .eq. 'y') then
            tang = ang
        else
            tang = full-ang
        endif
        len = get_len(insert, orig, tang)
        inst_p = get_inst_p(iso, rest_len, len)
c       { if the instantaneous iso-strength is negative, data invalid }
        if (inst_p .lt. 0.0) then
            call errstr(tang)
            goto 200
        endif
        forin = get_forin(constc, insert, vel, vmax, inst_p)
c       { external torque is dependent on angle between limbs }
        torq = (sin(tang))*forin
    
```

Strength Modeling

200

```
      write (unit=5,fmt='(f7.2,10x,f7.2,10x,f7.2,10x,f7.2,10x,f5.3)')  
2      tang,len,forin,torq,torq/forin  
      continue  
      ang = ang+interv  
end do  
close (unit=5)  
end
```


Strength Modeling

```

program isokisop

character*75 fnam, mus
character ext
real rest len, insert, orig, cross, funit, vmax
real constc, starta, enda, full, vel, tim, torq, iso
common /fbl/ fnam, mus, ext
common /sbl/ rest len, insert, orig, cross, funit, vmax
common /thbl/ constc, starta, enda, full, vel, tim, torq, iso

real len, forin, ang, tang
real interv
logical check

c { program to obtain strength data with Hill's equation }
c {   using isometric strength at rest length           }

100  continue
    call inparam

c { check values }
    call wrchk(check)
    if (.not. check) go to 100

c { values okay - open file and calculate values }
    open (unit=5, file=fnam, status='new', carriagecontrol='list')
    call dtor(starta, enda, full)
    interv = vel*tim
    ang = starta
    call wrstat
    call wrlab
    do while (ang .le. enda)
        if (ext .eq. 'y') then
            tang = ang
        else
            tang = full-ang
        endif
        len = get len(insert, orig, tang)
        forin = get forin(constc, insert, vel, vmax, iso)
        torq = (sin(tang))*forin
        write (unit=5, fmt='(f7.2,10x,f7.2,10x,f7.2,10x,f7.2,10x,f5.3)')
2          tang, len, forin, torq, torq/forin
        ang = ang+interv
    end do
    close (unit=5)
end

```

Strength Modeling

```
subroutine wrstat
include 'defparam.inc'
```

```
c { subroutine to write parameter values to data file }

write (unit=5,fmt='(2a)') ' Output data file: ',fnam
if (ext .eq. 'y') then
    write (unit=5,fmt='(2a)') ' Extension of ',mus
else
    write (unit=5,fmt='(2a)') ' Flexion of ',mus
endif
write (unit=5,fmt=10) ' starting angle at ',starta,
2 ' radians'
10 format (a,f7.2,a)
write (unit=5,fmt=10) ' ending angle at ',enda,' radians'
write (unit=5,fmt=10) ' full range is ',full,' radians'
write (unit=5,fmt=10) ' rest length is ',rest len,' cm'
write (unit=5,fmt=10) ' insertion length is ',insert,' cm'
write (unit=5,fmt=10) ' origin length is ',orig,' cm'
write (unit=5,fmt=10) ' cross-sectional area is ',cross,
2 ' sq-cm'
write (unit=5,fmt=10) ' force per unit area is ',funit,
2 ' N/sq-cm'
write (unit=5,fmt=10) ' constant c is ',constc
write (unit=5,fmt=10) ' max velocity is ',vmax,' rad/s'
write (unit=5,fmt=10) ' velocity of contraction is ',vel,
2 ' rad/s'
write (unit=5,fmt=10) ' load is ',torq
write (unit=5,fmt='(a,f4.3,a)') ' time interval is ',tim,' s'
return
end

subroutine wrlab
include 'defparam.inc'

c { subroutine to write header for data in data file }

write (unit=5,fmt='(a)') ' ',' ',' '
write (unit=5,fmt='(a,7x,a,10x,a,10x,a)') ' angle(rad)','muscle',
2 'muscle','external'
write (unit=5,fmt='(a,3x,a,5x,a,8x,a,5x,a)') 'between limbs ',
2 'length(cm)','force(N)','torque(N-cm)','torque/force'
write (unit=5,fmt='(a)') ' '
return
end
```

Strength Modeling

```

subroutine wrchk (check)
include 'defparam.inc'
logical check
character ans

```

c { subroutine to confirm data recorded }

```

check = .false.
write (unit=*,fmt='(a)') ' These are the values recorded: '
write (unit=*,fmt='(a)') ' '
write (unit=5,fmt='(2a)') ' Output data file: ',fnam
if (ext .eq. 'y') then
    write (unit=5,fmt='(2a)') ' Extension of ',mus
else
    write (unit=5,fmt='(2a)') ' Flexion of ',mus
endif
write (unit=5,fmt=10) ' starting angle at ',starta,
2 ' degrees'
10 format (a,f7.2,a)
write (unit=5,fmt=10) ' ending angle at ',enda,' degrees'
write (unit=5,fmt=10) ' full range is ',full,' degrees'
write (unit=5,fmt=10) ' rest length is ',rest_len,' cm'
write (unit=5,fmt=10) ' insertion length is ',insert,' cm'
write (unit=5,fmt=10) ' origin length is ',orig,' cm'
write (unit=5,fmt=10) ' cross-sectional area is ',cross,
2 ' sq-cm'
write (unit=5,fmt=10) ' force per unit area is ',funit,
2 ' N/sq-cm'
write (unit=5,fmt=10) ' constant c is ',constc
write (unit=5,fmt=10) ' max velocity is ',vmax,' rad/s'
write (unit=5,fmt=10) ' velocity of contraction is ',vel,
2 ' rad/s'
write (unit=5,fmt=10) ' load is ',torq
write (unit=5,fmt='(a,f4.3,a)') ' time interval is ',tim,' s'
write (unit=*,fmt='(a)') ' '
write (unit=*,fmt='(a,$)') ' Are the values correct-y/n? '
read (unit=*,fmt='(a1)') ans
if (ans .eq. 'y') then
    check = .true.
endif
return
end

```

Strength Modeling

```
subroutine dtor (starta,enda,full)
real vel,starta,enda,full
real pi
```

```
c { subroutine to convert degrees to radians }
```

```
pi = (3.14/180.0)
starta = starta*pi
enda = enda*pi
full = full*pi
return
end
```

```
real function get_inst_p (iso,rest_len,len)
real rest_len,len
real temp,iso,tense
```

```
c { function to obtain the instantaneous isometric strength }
c { based on Stern's equation }
```

```
temp = 100*(len/rest_len)
if (temp .le. 107.88) then
    tense = (log10(temp)-2)**2
    get_inst_p = (100-(tense/0.00054448))/100
else
    get_inst_p = (212.77-(1.0638*temp))/100
endif
get_inst_p = (get_inst_p)*iso
return
end
```

Strength Modeling

```
subroutine inparam
include 'defparam.inc'
character ans
```

```
c { routine to obtain parameter values }

write (unit=5,fmt=10) ' Output file: '
10 format (a,$)
read (unit=5,fmt=20) fnam
20 format (a)
write (unit=5,fmt=10) ' Muscle to be tested: '
read (unit=5,fmt=20) mus
write (unit=5,fmt=10) ' Extension - y/n? '
read (unit=5,fmt='(a1)') ext
print *, 'enter rest length, insertion length, and origin length in cm'
read (unit=5,fmt=30) rest_len
30 format (f7.2)
read (unit=5,fmt=30) insert
read (unit=5,fmt=30) orig
print *, 'enter cross-sectional area (cm), force/sq-cm (kg/sq-cm), and'
print *, ' maximum velocity (rad/s)'
read (unit=5,fmt=30) cross
read (unit=5,fmt=30) funit
read (unit=5,fmt=30) vmax
write (unit=5,fmt=10) ' Value of constant, c? '
read (unit=5,fmt='(f4.2)') constc
print *, 'enter starting angle and end angle in degrees'
read (unit=5,fmt='(f6.2)') starta
read (unit=5,fmt='(f6.2)') enda
write (unit=5,fmt=10) ' Full range of movement, degrees? '
read (unit=5,fmt=30) full
write (unit=5,fmt=20) ' isokinetic or isotonic? '
write (unit=5,fmt=10) ' type "k" for isokinetic and "t" for
2 isotonic: '
read (unit=5,fmt='(a1)') ans
if (ans .eq. 'k') then
    write (unit=5,fmt=10) ' Velocity of contraction, rad/s? '
    read (unit=5,fmt=30) vel
else
    write (unit=5,fmt=10) ' load, kg? '
    read (unit=5,fmt=30) torq
endif
write (unit=5,fmt=10) ' time interval for calculations,s? '
read (unit=5,fmt='(f4.3)') tim
iso = cross*funit
return
end
```

Strength Modeling

```
real function get_forin (constc,insert,vel,vmax,inst_p)
real constc,insert,vel,inst_p,vmax
real fact
```

```
c { function to obtain the force of muscular contraction }
```

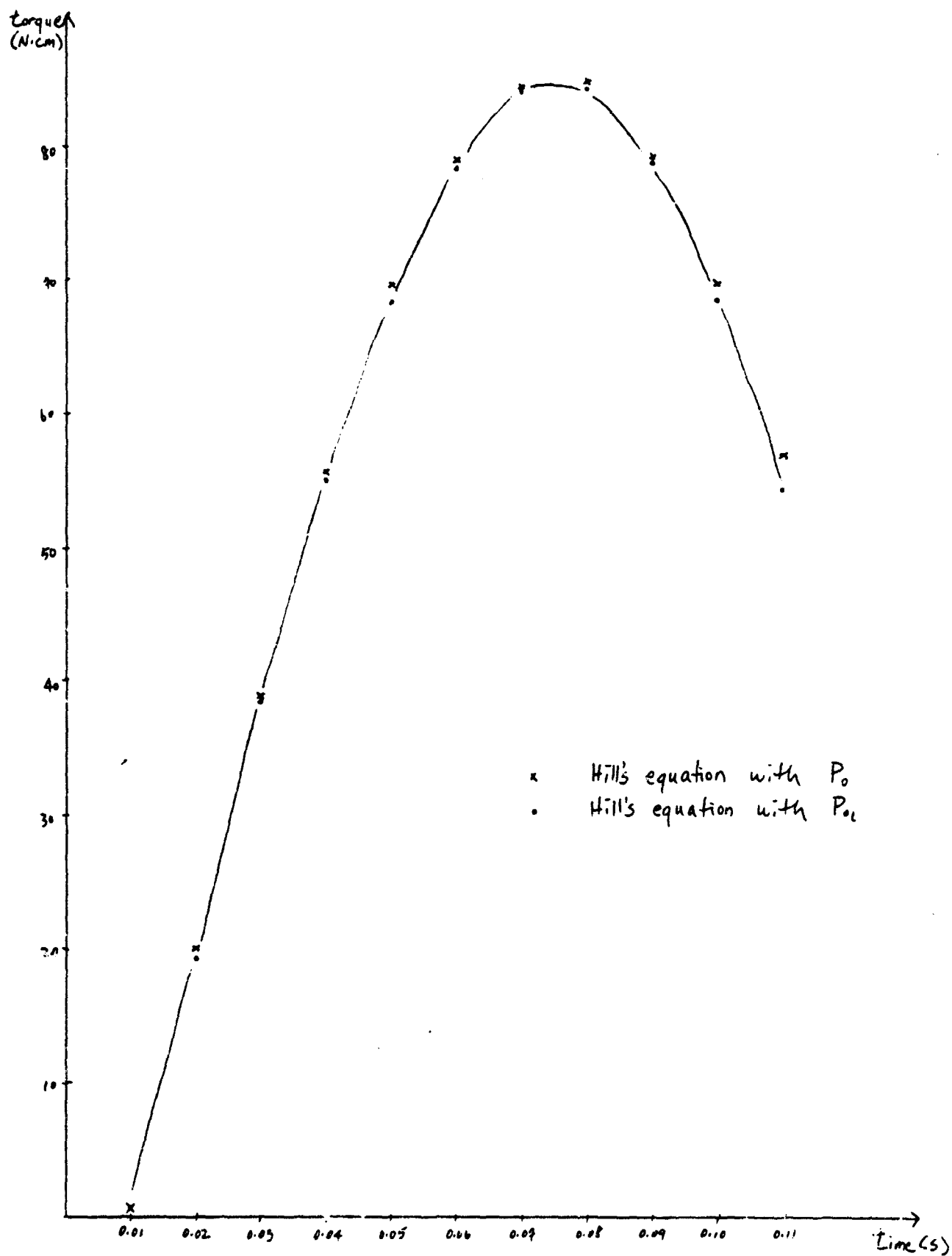
```
fact = ((1+constc)*insert*vel)/((insert*vel)+(constc*vmax))
get_forin = inst_p*(1+fact)
return
end
```

```
subroutine errstr (tang)
real tang
```

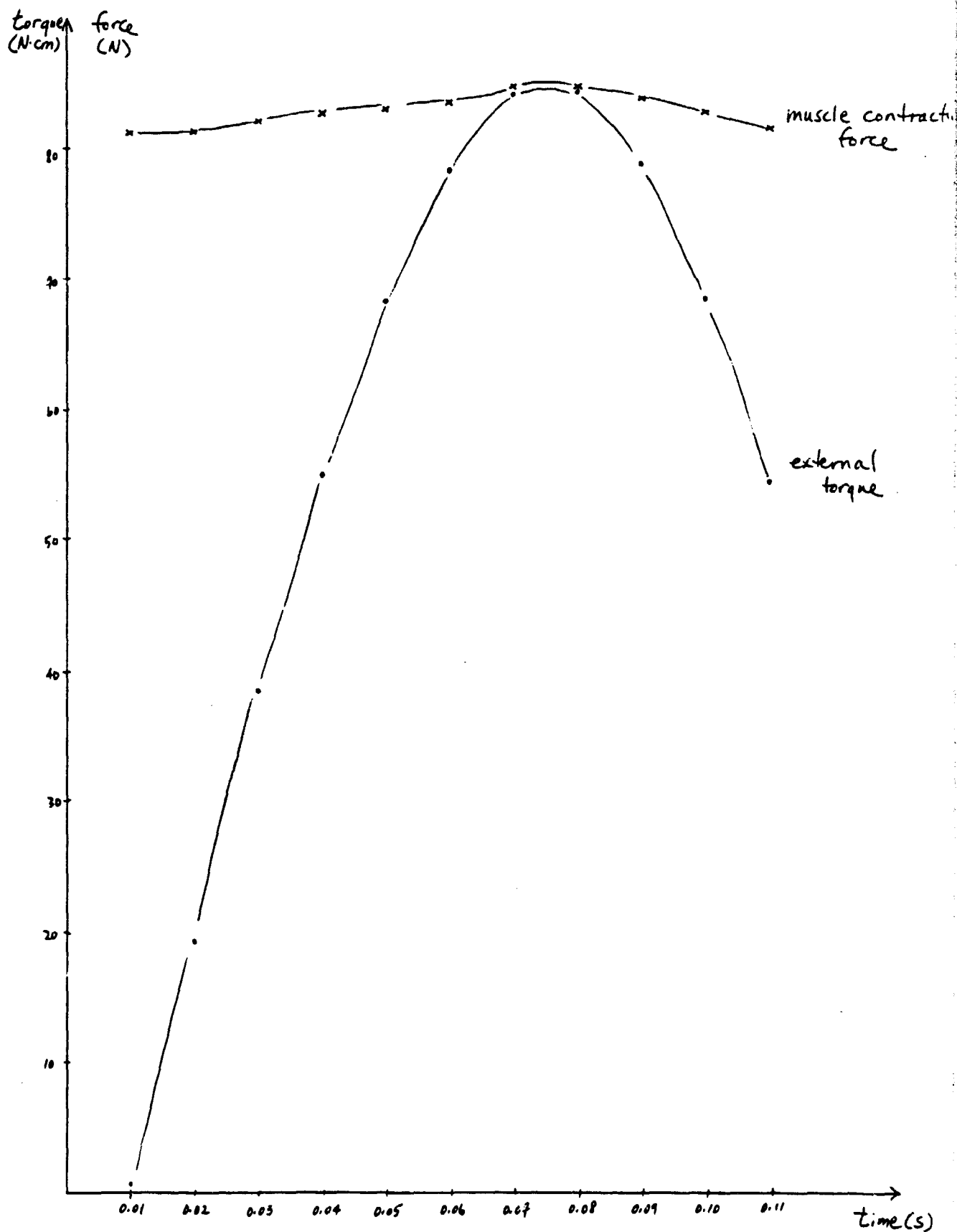
```
c { subroutine to give error message when strength data invalid }
```

```
20 write (unit=5,fmt=20) ' strength not valid at ',tang,' radians'
format (a,f7.2,a)
write (unit=*,fmt=20) ' strength not valid at ',tang,' radians'
return
end
```

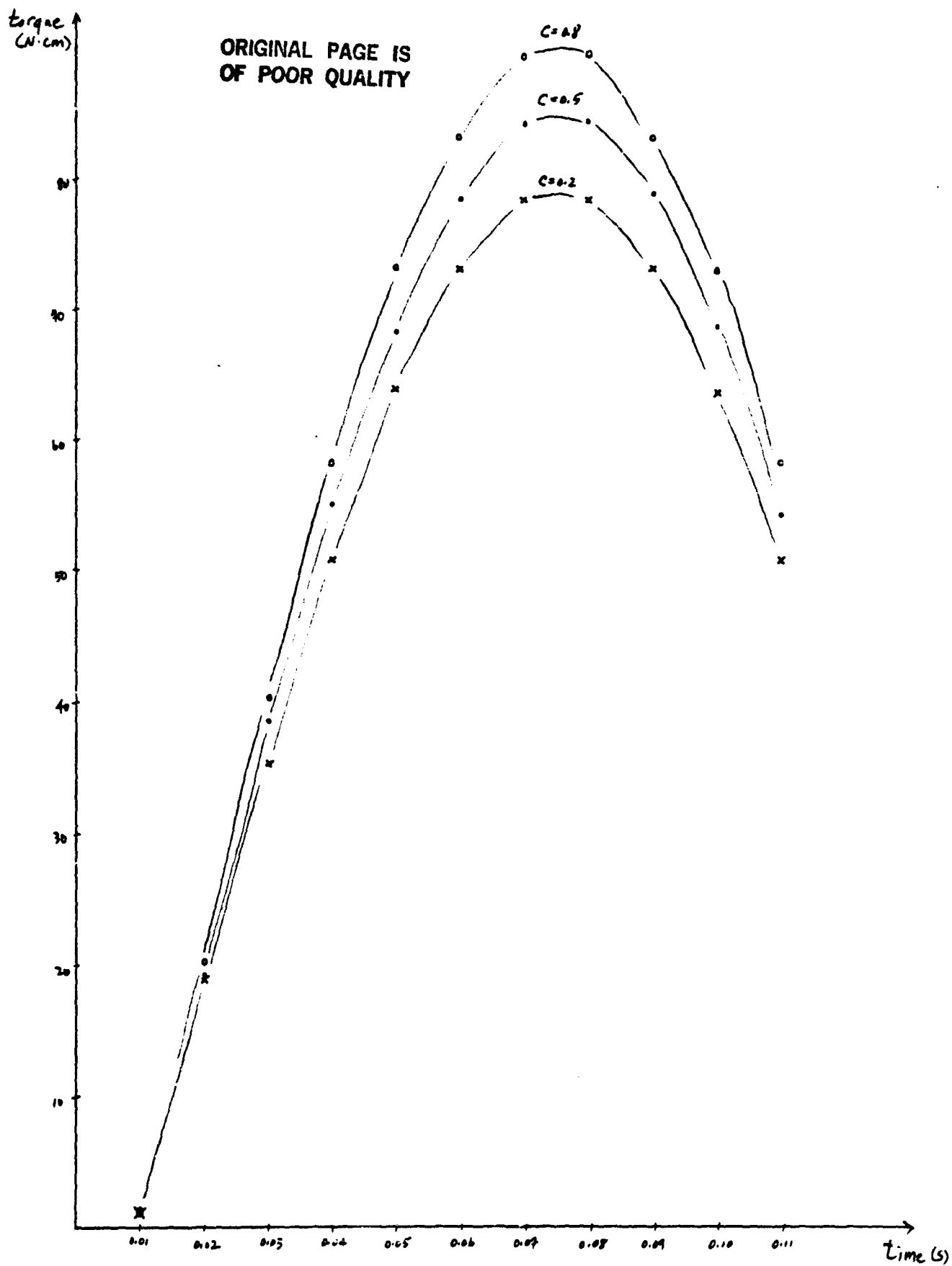
Graph 1: Comparison of Hill's Equation with P_0 and with P_{0L} .



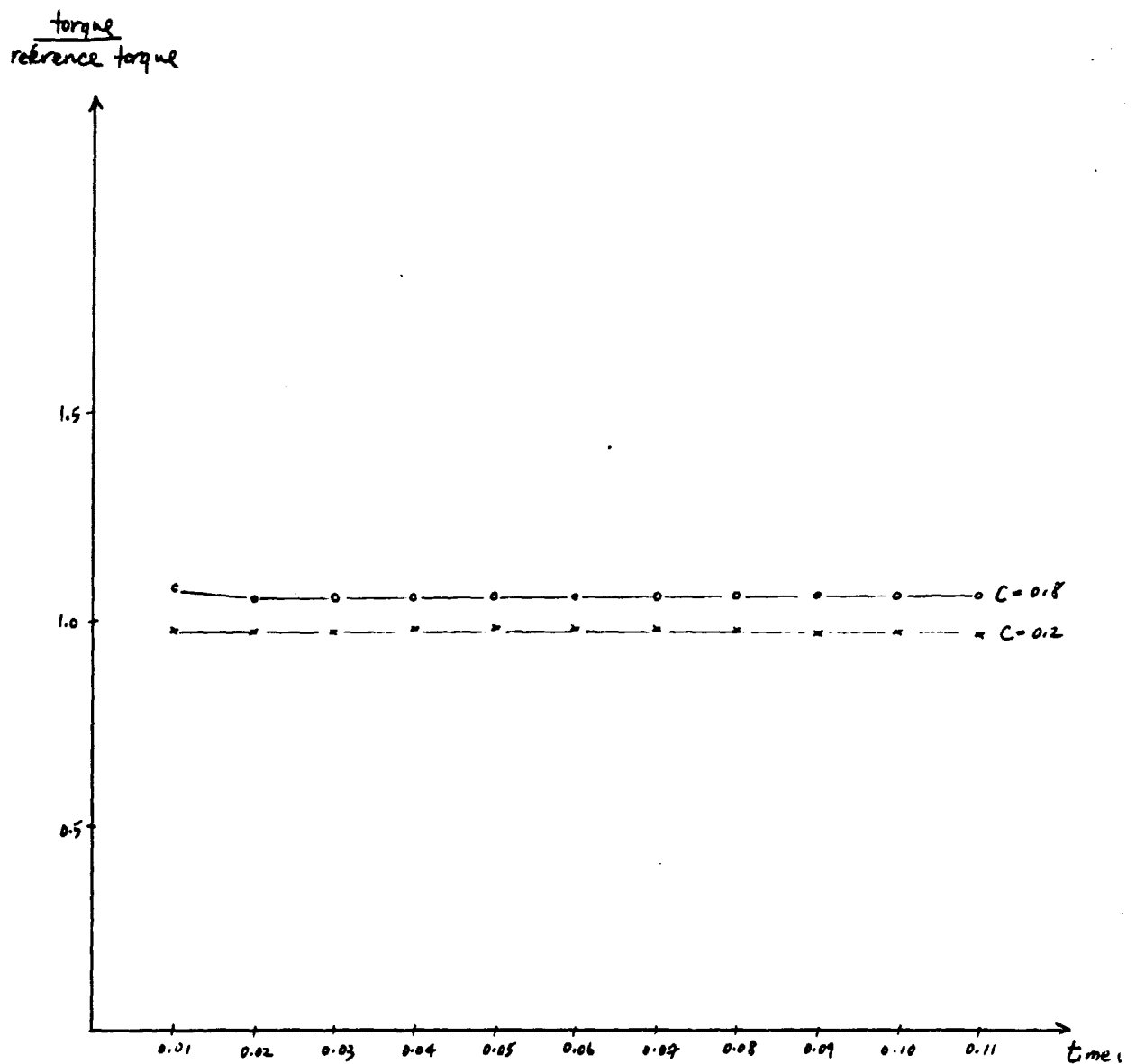
Graph 2: Comparison of Internal Muscle Force and External Torque.



Graph 3: Resultant Torque at Different Values of C.

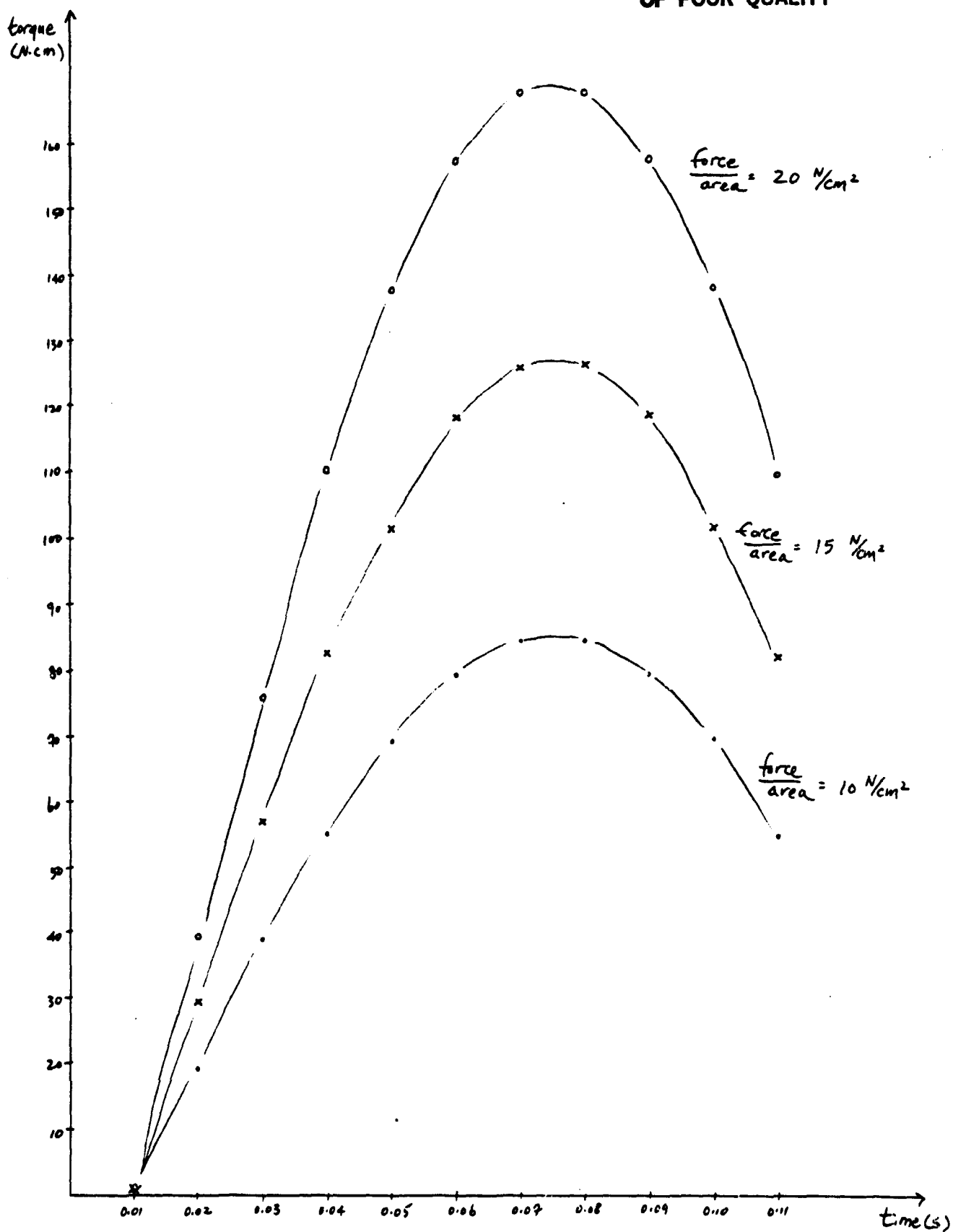


Graph 3a: Normalized Torque for Different Values of C.

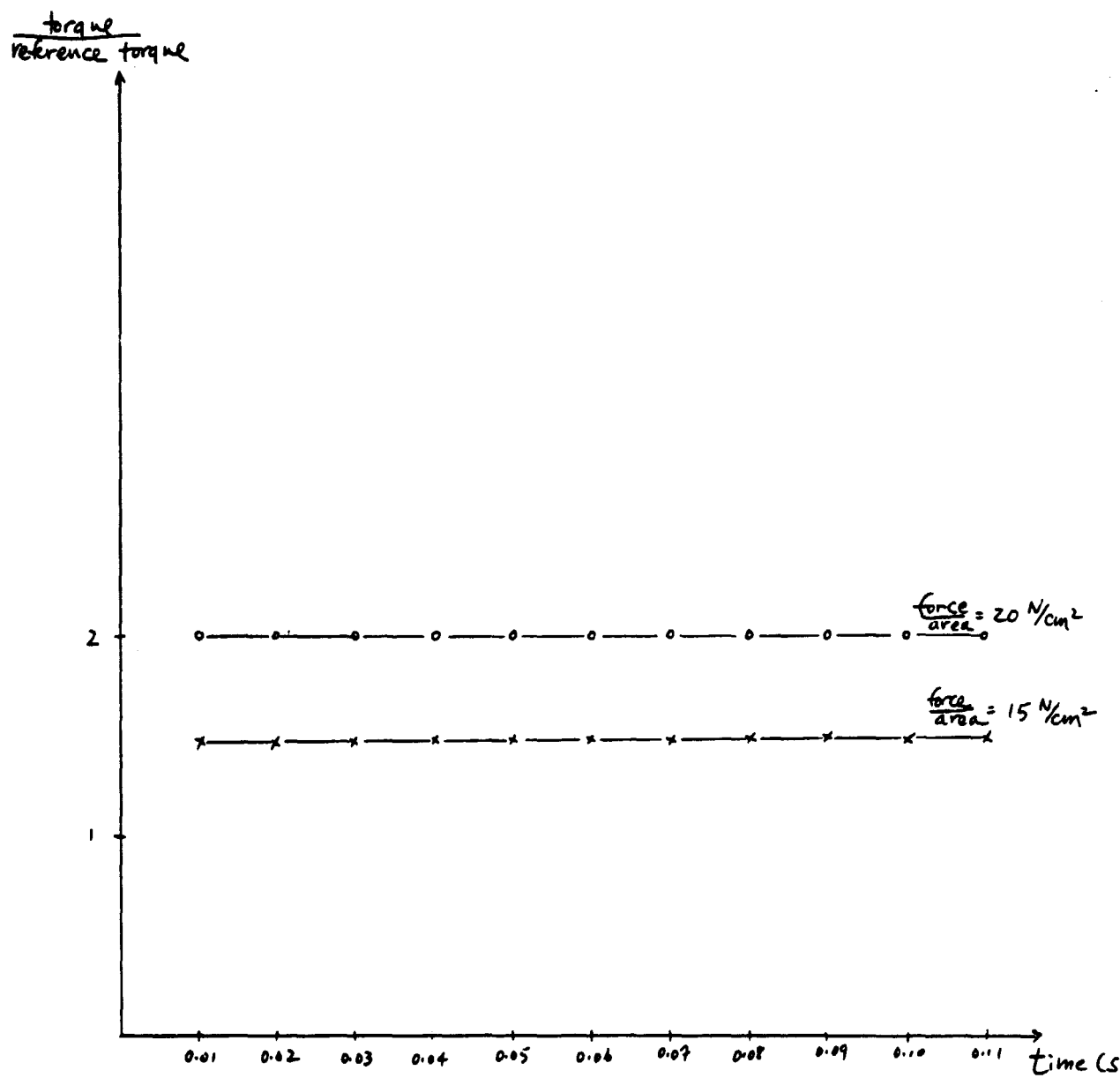


Graph 4: Resultant Torque for Different Values of Force/Area.

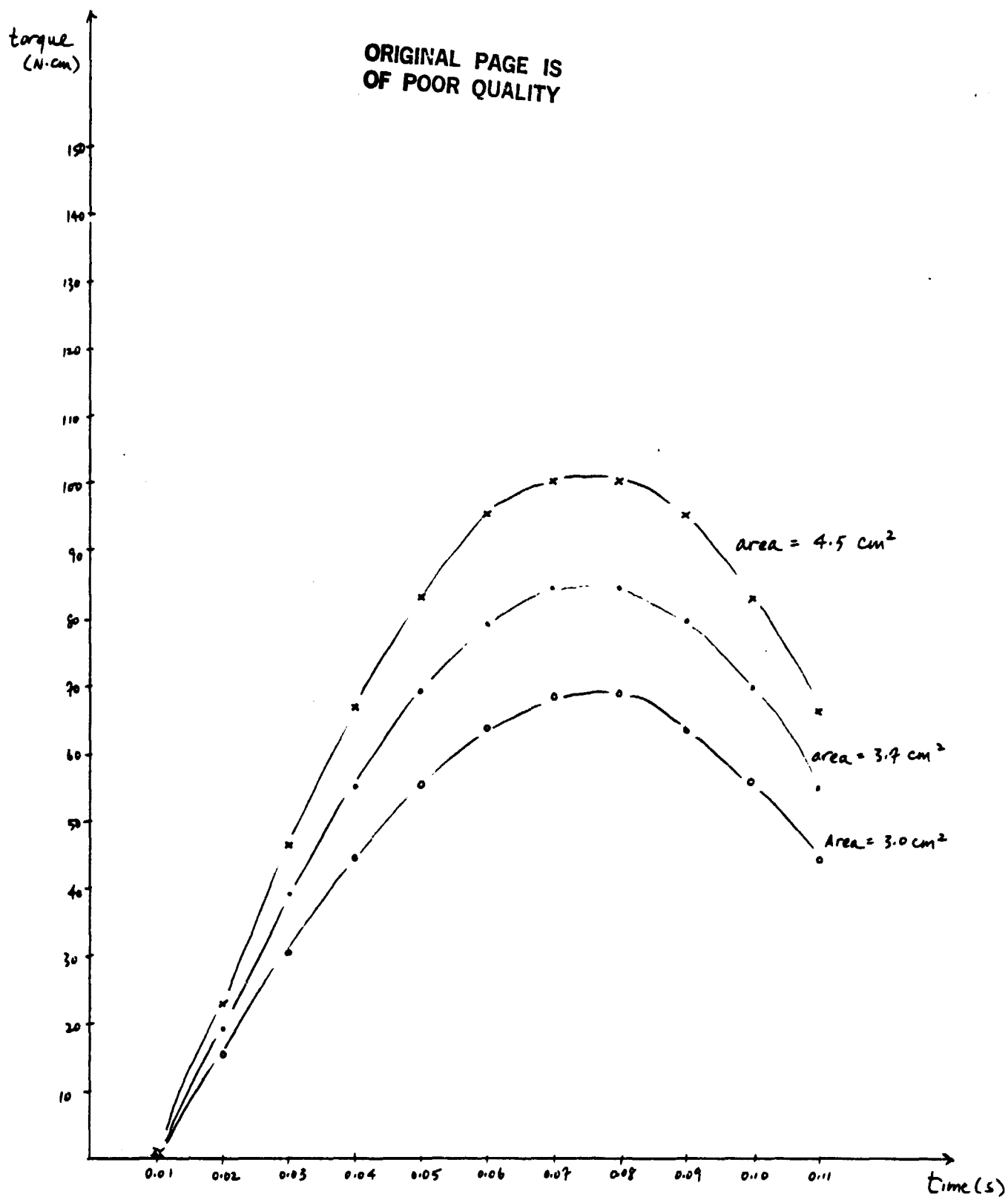
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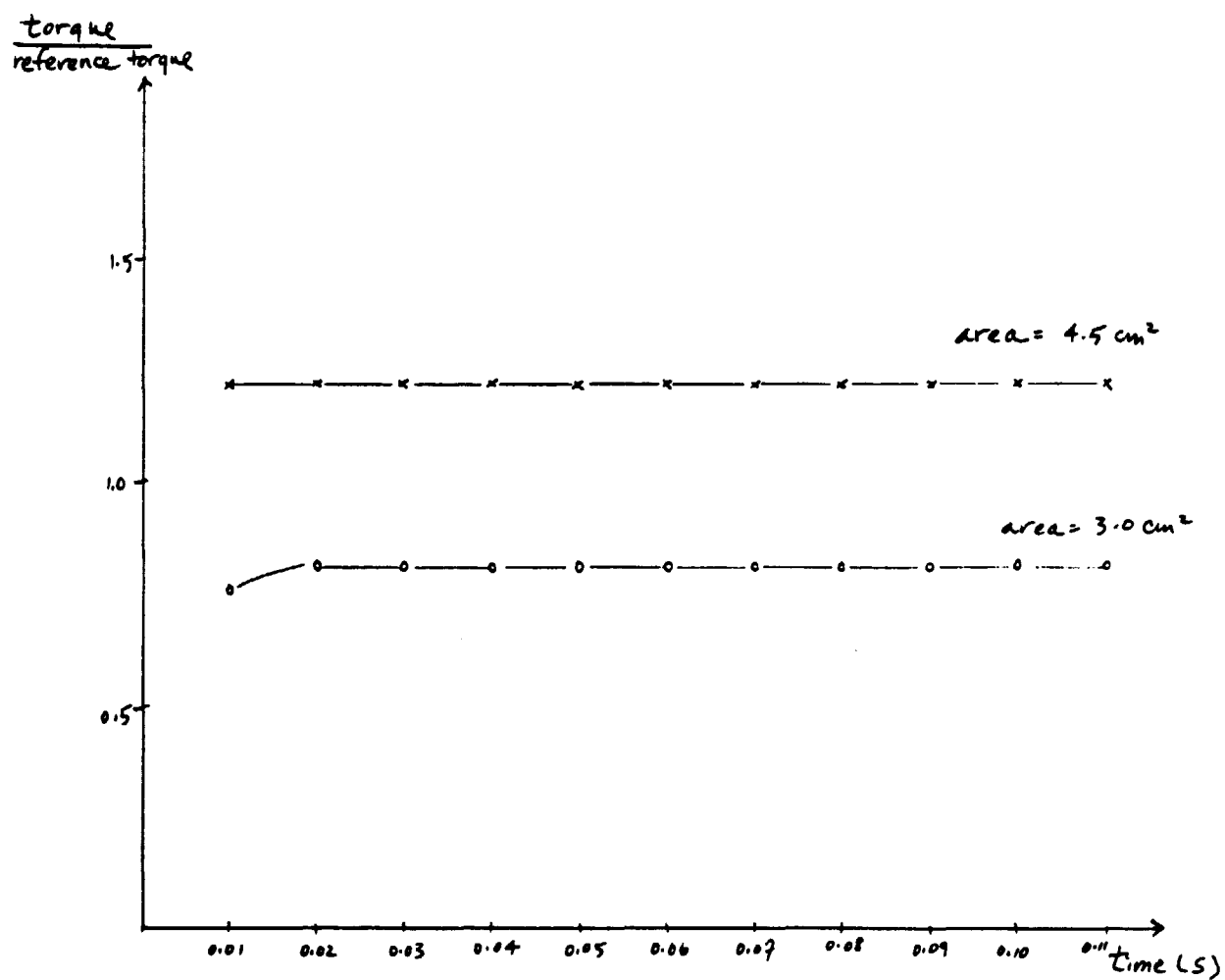
Graph 4a: Normalized Torque for Different Values of Force/Area.



Graph 5: Resultant Torque for Different Values of Cross-Sectional Area.

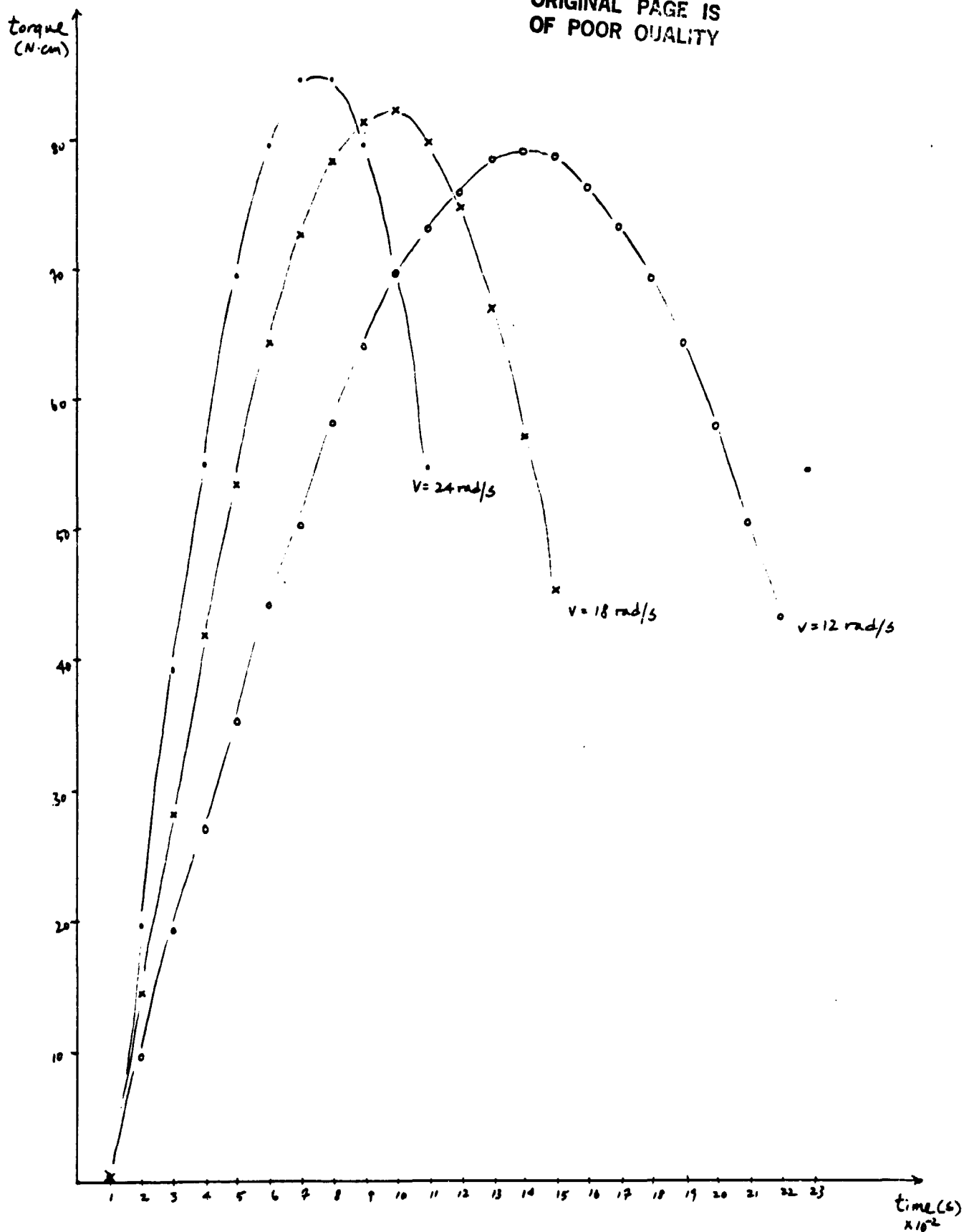


Graph 5a: Normalized Torque for Different Values of Cross-Sectional Area.



Graph 6: Resultant Torque for Different Values of V.

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Graph 6a: Torque Relative to Peak Torque for Different Values of V.

